Analysis of the Efficiency of Public Spending on Public Security in the Federative States of Brazil

Análise da Eficiência dos Gastos Públicos com Segurança Pública nos Estados Federativos do Brasil
Análisis de la Eficiencia de los Gastos Públicos en Seguridad Pública en los Estados Federativos de Brasil.

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Abstract

Research objective: Analyze the efficiency of public resource allocation in the domain of public security across the federative states of Brazil.
Theoretical framework: The theoretical foundation drew upon studies on public security and the efficiency of public resource allocation. The theoretical approach underscores the interconnectedness between the efficiency of public resource management and the promotion of public security, emphasizing the importance of an integrated and effective approach to address social and security issues.
Methodology: A Data Envelopment Analysis (DEA) model was used to assess the efficiency of public resource allocation in public security in the 26 Brazilian federative states and the Federal District from 2014 to 2021. A BCC-output (Banker, Charnes, and Cooper) model was applied and the Malmquist Index to enable the observation of how efficiency evolved over time. Per capita spending on public security served as the input, while Intentional Lethal Violent Crimes were used as outputs.
Results: Research findings reveal an increase in the efficiency of public resource allocation in public security services, as evidenced by the MI. The study indicates that performance improved during the study period, initially influenced by technical efficiency in the early years and increasingly by technological efficiency from 2017 to 2018, a year marked by a significant reduction in violence in Brazil (Lima et al., 2022).
Originality: The study examines the behavior of the efficiency of public resource allocation in public security from 2014 to 2021, using the Malmquist Index, providing insights into hypotheses related to variations in crime rates.
Theoretical and practical contributions: This study provides empirical evidence regarding the efficiency of public spending on public security and underscores the potential for managerial enhancements without the need for budget increases.
Keywords: Public Security. Efficiency. Public Management. DEA. Malmquist Index

Resumo

Objetivo da pesquisa: Analisar a eficiência da aplicação dos recursos públicos com segurança pública dos estados brasileiros.
Enquadramento teórico: Como base teórica foi utilizado estudos sobre segurança pública e eficiência da aplicação dos recursos públicos. A abordagem teórica destaca a interconexão entre a eficiência na gestão de recursos públicos e a promoção da segurança pública, enfatizando a importância de uma abordagem integrada e eficaz para enfrentar os desafios sociais e de segurança.
Metodologia: Utilizou-se um modelo de Análise Envoltória de Dados para mensurar a eficiência da aplicação dos recursos públicos com segurança pública dos 26 estados brasileiros e do Distrito Federal no período de 2014 a 2021. Foi utilizado um modelo BCC-*output* e aplicou-se o Índice de Malmquist para verificar a evolução da eficiência ao longo do tempo. Como *input* foi utilizado a Despesa *per capita* realizada com Segurança Pública e como *outputs* foram utilizados os Crimes Violentos Letais Intencionais.

Resultado: Os resultados da pesquisa mostram um incremento na eficiência da aplicação dos recursos públicos com segurança pública, demonstrado pelo IM onde verificou-se que o desempenho elevou-se no período estudado, sendo influenciado nos primeiros períodos pela eficiência técnica, e passando a ser mais influenciada pela eficiência tecnológica a partir do período de 2017 a 2018, ano que há uma queda acentuada na violência no Brasil (Lima et al., 2022).
Originalidade: O trabalho analisa o comportamento da eficiência da aplicação dos recursos públicos com segurança pública no período de 2014 a 2021 a partir do uso do Índice de Malmquist, apresentando *insigths* sobre hipóteses relacionadas as variações dos índices de criminalidade
Contribuições teóricas e práticas: Esta pesquisa fornece evidências empíricas sobre a eficiência dos gastos públicos em segurança pública e destaca o potencial de melhorias gerenciais sem a necessidade de aumentos orçamentários.
Palavras chave: Segurança Pública. Eficiência. Gestão pública. DEA. Índice de Malmquist

Resumen

Objetivo de la investigación: Analizar la eficiencia de la aplicación de recursos públicos en la seguridad pública de los estados brasileños.
Marco teórico: Se utilizó como base teórica estudios sobre seguridad pública y eficiencia en la aplicación de los recursos públicos. El enfoque teórico destaca la interconexión entre la eficiencia en la gestión de los recursos públicos y la promoción de la seguridad pública, enfatizando la importancia de un enfoque integrado y eficaz para enfrentar los desafíos sociales y de seguridad.
Metodología: Se utilizó un modelo de Análisis Envolvente de Datos para medir la eficiencia en la aplicación de recursos públicos en seguridad pública en los 26 estados brasileños y en el Distrito Federal durante el período de 2014 a 2021. Se empleó un modelo BCC-output y se aplicó el Índice de Malmquist para verificar la evolución de la eficiencia a lo largo del tiempo. Como *input* se consideró el Gasto *per cápita* realizado en Seguridad Pública, y como *output* se utilizaron los Crímenes Violentos Letales Intencionais.
Resultados: Los resultados de la investigación muestran un aumento en la eficiencia en la aplicación de los recursos públicos con la seguridad pública, demostrado por el IM donde se encontró que el desempeño aumentó en el período estudiado, siendo influenciado en los primeros períodos por la eficiencia técnica, y volviéndose más influenciado debido a la eficiencia tecnológica de 2017 a 2018, año en el que se produjo una marcada disminución de la violencia en Brasil (Lima et al., 2022).
Originalidad: El trabajo analiza el comportamiento de la eficiencia en la aplicación de los recursos públicos en seguridad pública en el período de 2014 a 2021 mediante el uso del Índice de Malmquist, presentando perspectivas sobre hipótesis relacionadas con las variaciones de los índices de criminalidad
Aportes teóricos y práticos: Esta investigación proporciona evidencia empírica sobre la eficiencia del gasto público en seguridad pública y destaca el potencial de mejoras de gestión sin la necesidad de aumentos presupuestarios.
Palabras clave: Seguridad Pública. Eficiencia. Gestión pública. DEA. Índice de Malmquist.

1. Introduction

Public security is an important prerequisite for ensuring economic and social well-being, as well as the sustainable development of states (Zou et al., 2021). Cities that efficiently implement public security measures demonstrate greater resilience to social emergencies and are able to maintain a dynamic balance while promoting coordinated development, thus ensuring the protection of the environment, society, and individuals (Lu et al., 2016; Zou et al., 2021).

The increase in crime rates and the associated problems, along with the growing sense of insecurity, pose obstacles to the political consolidation of democracy in Brazil (Brazilian Forum of Public Security – FBSP, 2022). Therefore, public security is a fundamental function of the government for preventing and managing various types of incidents, catastrophes, and crises to safeguard the lives and property of its citizens (Zou et al., 2021) and to ensure the sustainable development outlined in Sustainable Development Goals (SDG) 16 (UN, 2015).

According to the data system of the United Nations Office on Drugs and Crime (UNODC), Brazil has the highest absolute number of homicides in the world (FBSP, 2022). Despite having only 2.7% of the world's population, Brazil accounts for 20.5% of homicides. The total number of murders in 102 countries is 232,676, while in Brazil, it stands at 47,722 for the year 2021 (FBSP, 2022). The objective of society is to minimize the harm caused by crime, and this depends on how public officials allocate public resources (Odon, 2018).

One commonly debated issue in academia is the inefficiency in utilizing available resources (Santos & Rover, 2019). Santos and Rover (2019) explain that increased allocation of public resources across various government functions is crucial for improving development and reducing existing disparities. Public budgeting plays a crucial role in organizing and allocating resources for government planning.

Through the budget, actions are established, goals are set, agents responsible for execution are determined, and resources are allocated to balance the population's needs with the available resources (Santos et al., 2021). In this context, resource allocation has to be analyzed to enable an assessment of how public security is managed by the public administration, thus provide greater benefits to society.

Schull et al. (2014) emphasized that evaluating the efficiency of public spending in the field of security is an important indicator so that managers can apply them optimally, aiming for greater benefits to society. Efficiency is related to the best use of available resources and is directly linked to rationality and productivity because efficiency is achieved by producing more utility while demanding the same number of resources (Santos & Rover, 2019). Efficiency is achieved through the maximal utilization of existing resources to meet the needs and desires of individuals and organizations. The resources allocated to public security are expected to result in improvements in security performance (Zou et al., 2021).

In efficiency studies, Data Envelopment Analysis (DEA) is one of the most used tools, particularly concerning the allocation of public resources (Lampe & Hilgers, 2015; Ratner et al., 2023). However, efficiency scales at different time periods are not comparable. The Malmquist Index is used to analyze dynamic efficiency over time (Zou et al., 2021).

Therefore, this study aims to analyze the efficiency of public resource allocation in public security services in the Brazilian states and the behavior of this efficiency over time. For this purpose, a model was designed using DEA to measure the efficiency of public resource allocation in public security services in the Brazilian states and the Federal District from 2014 to 2021. The Malmquist Index was also calculated to assess the behavior of this efficiency over time.

For Freitas Júnior et al. (2020), the public sector has increased its spending on public security, but there has been no significant improvement in crime indicators. Thus, this study becomes relevant by showcasing the evolution of the efficiency of Brazilian federative units over time, which can assist public officials in decision-making. Studying the period from 2014 to 2021 and applying the Malmquist Index is also a contribution of this work. The time frame covered is significant, during which changes in public policies, societal demands, and economic conditions may have occurred. Evaluating efficiency over this period allows for the identification of trends and variations in resource allocation.

Another noteworthy contribution of this study is that its results can be used to underpin public security policies. Identifying areas where efficiency can be improved can lead to more effective policies and a more efficient allocation of resources. The present study serves the public interest, as it focuses on a sector that is essential to the quality of life of the population.

2 Theoretical Framework

2.1 Public Security

Brazilian public security has numerous deficiencies that need to be overcome, despite its evolution over time, constantly adapting to the surrounding circumstances (Freitas Júnior et al., 2021). It can be said that the topic of public security is one of the most discussed by society and public administrators, thus bringing to light the critical context that affects the population (Freitas Júnior et al., 2020).

There are economic, social, and political impacts of criminality, including costs in the healthcare system, loss of productivity, and reduced well-being (Hoeffler, 2017). The economic impact is enhanced by crime intensity, which creates particular conditions in a given locality; it is, therefore, closely tied to the economic context of such territory.

From a social perspective, the impact of criminality may jeopardize the quality of life of citizens and society. Politically, policies and strategies need to be developed to address this issue (Monte & Leopoldino, 2020). Moreover, level of criminality is associated with various factors, such as high unemployment, high interest rates, low expectations of social mobility, and increased inflation (Fernandes Júnior et al., 2017; Monte & Leopoldino, 2020).

It is understood that the conception and implementation of public security policies are more complex and cross-cutting than mere actions of public institutions through mechanisms of social control based on the legal use of force (Freitas Júnior et al., 2020). The authors also emphasized the need for a new conception of public security that reaches across the entire country, encompassing all aspects of security, whether in the economic realm or in social and cultural aspects, consequently increasing the potential for achieving a fairer citizenship.

It is worth noting that the increase in crime rates is associated with the state's inability to deal with the public security problem (Scalco, 2007). Scalco (2007) questioned whether this inability stems from the inefficiency of public policies adopted to combat crime, whether it is associated with factors unrelated to the presence of the state, or whether it is related to both.

The effectiveness of public security policy is not solely tied to increased investments in the sector, but rather to the strategic allocation of resources towards actions that yield medium- to long-term impact (Figueiredo et al., 2021). The authors further suggest that social, economic, and cultural factors are intertwined with crime control.

Freitas Júnior et al. (2020) highlight several factors that may explain the efficiency of public spending in public security, including education, social inequality, unemployment, police force effectiveness, and others.

The Brazilian Public Security Yearbook of 2022 shows that 2017 was the year when Brazil had the highest number of violent and intentional deaths (FBSP, 2022). As of 2018, this indicator has been decreasing. From 2020 to 2021, there was a 6.5% decrease in this indicator. There are various hypotheses in the literature regarding the decrease in intentional violent deaths in Brazil, but the yearbook itself highlights four of them: demographic changes, targeted violence prevention policies and police integration models, the action of organized crime, and gun control and recent legislative changes. These arguments are explored in more detail in Section 4.4 of this work.

2.2 Efficient Application of Public Resources

Efficiency is related to the methods employed to achieve a goal, given a certain number of resources. It involves comparing what has been produced with the maximum that could be produced with the same number of resources, in other words, producing more with less. Efficiency pertains to maximizing the efficient use of resources and minimizing waste. Thus, efficiency is attained when existing resources are optimally utilized to meet the needs and desires of individuals and organizations (Santos & Rover, 2019).

Public Administration, both in Brazil and worldwide, has undergone transformations, transitioning from a bureaucratic model to managerial models, with the New Public Management (NPM) as its exemplar, and more recently, to the New Public Governance (NPG) (Osborne, 2006). When comparing NPM with NPG, efficiency is observed as a common element between the two models of public administration. With the goal of ensuring that society has broader access to the goods and services offered by the State, Costa et al. (2015) emphasized the need for these resources to be allocated effectively and, most importantly, efficiently (Costa et al., 2015; Rover & Santos, 2019). The significance of maintaining efficiency in the use of public resources is underscored owing to the considerable opportunity costs associated with delivering a specific service (Costa et al., 2015). Therefore, this constant pursuit of efficiency becomes an essential priority for the well-being of society (Pereira Filho et al., 2010).

For public administration to succeed, more economical and viable means are needed. Efficiency should not only refer to the reduced use of resources but also to meeting the needs of society. It should be clear how objectives were achieved and how the resources paid by taxpayers were spent.

According to Santos and Rover (2019), evaluating efficiency in the public sphere is essential and should be adopted by public managers, considering that the impact of investments in different sectors should be reflected in quality services for the population, and improved indicators.

Efficiency as the optimal combination of the inputs and methods needed for the production process to generate the maximum possible outputs. This means that efficiency is the ability to do things efficiently, i.e., to minimize the input-output relationship. It aims to ensure the optimization of resource utilization and, therefore, it relates to the means rather than the ends (Santos et al., 2018; Santos et al., 2020).

The relationship between inputs and outputs reveals whether the production process of goods and services offered by the government is being conducted efficiently with the ideal consumption of certain physical quantities of inputs (technical efficiency). This relationship allows analyzing to what extent the government can increase its efficiency and reduce spending pressure, positively affecting its financial condition (Santos et al., 2020).

Performance indicators allow the evaluation of the application and results of public resource utilization, thus assisting managers in the decision-making process by reconciling efficient spending and quality of services. This benefits society, which can and should demand responsible actions from managers, encouraging the balanced allocation of public resources (Santos & Rover, 2019).

Attention to public security, combined with a national-level policy, has a significant impact on crime prevention, especially because each city feels the impact of this phenomenon differently (Bohn et al., 2015). Meeting this objective should, therefore, be a duty of the government, and efficiency analysis can serve as a good guide (Santos et al., 2021).

According to Odon (2018), society's objective is to minimize the harm caused by crime and, consequently, to persuade individuals not to commit crimes, ideally. This approach is closely linked to how agents and politicians distribute and invest public resources.

Furthermore, the high incidence of crime is attributed to the inefficiency of public security policies in curbing violence and crime, constituting a complex phenomenon intertwined with various other factors, such as unemployment and the absence of policies that promote equality and social inclusion (Figueiredo et al., 2021).

These considerations underscore the importance of studies dedicated to evaluating the efficiency of public resource allocation in promoting public security for the benefit of society. In this context, efficient public management is considered efficient when it can achieve the results established in its goals and objectives while using fewer resources, which is reflected in the optimization of converting inputs into quality products and services offered to the population. The analysis of efficiency in the application of public resources in public security contributes to more effective management and the maximization of social well-being, allowing for the appropriate use of available resources for the benefit of society (Souza et al., 2012; Santos & Rover, 2019).

The Data Envelopment Analysis (DEA) and the Malmquist Productivity Index are commonly employed in various studies examining public sectors such as education, health, and housing, among others (e.g., Mollahaliloglu et al., 2018; Oliveira et al., 2022; Santos & Rover, 2019), as well as in the field of public security (e.g., Lima & Marinho, 2017; Marzzoni, 2022; Pereira Filho, 2016; Zou et al., 2021).

3 Methodology

The first phase of this study involves measuring the efficiency scale of the allocation of public resources in the field of public security for the Brazilian Federal States and the Federal District for the years 2014 to 2021. A DEA-BCC model with output orientation is employed with the objective of constructing the efficiency scale of public expenditure on public security.

Data Envelopment Analysis (DEA) is a comprehensive and data-driven approach to performance and relative efficiency evaluation, treating each evaluated unit as an independent Decision-Making Unit (DMU) (Zou et al., 2021). It utilizes multiple input and output variables, which are calculated by comparing the efficiency of each DMU with the best-performing units (Zou et al., 2021). Since DEA is estimated based on a non-parametric methodology, there is no need to standardize indicator units, consider functional relationships between inputs and outputs, take into account pre-estimated parameters, or assume weights, thereby ensuring the integrity of the original information (Feng et al., 2014). Essentially, DEA provides a theoretical framework that does not require a weighting

procedure, making it the most highly regarded multicriteria evaluation method (Zou et al., 2021). Furthermore, among all methodologies for measuring efficiency, Lampe and Hilgers (2015) point out that DEA is widely used for assessing the efficiency of public expenditures.

3.1 Research Sample and Database

Brazil is divided into 5 major regions (North, Northeast, Midwest, Southeast, and South), which are further divided into 26 federative states plus the Federal District, totaling 27 federative units.

Therefore, the research sample consisted of the 26 Brazilian federative states plus the Federal District, totaling 27 federative units. It can be affirmed that the research sample is non-probabilistic because there was no random selection (Trochim et al., 2015).

The data used were collected in August 2022 and sourced from the Brazilian Institute of Geography and Statistics (BIGS), specifically from the National Household Sample Survey (PNAD). Information on public security expenditures was gathered from the National Treasury Secretariat (NTS) website in the Brazilian Public Sector Accounting and Fiscal Information System (Siconfi). Additionally, indicators from the ILVC for the years 2014 to 2021 were obtained from the Brazilian Public Security Yearbook (BPSY) website hosted by the Brazilian Forum on Public Security.

3.2 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a non-parametric approach employed to assess the relative efficiency of decision-making units operating with multiple inputs and outputs. (Bohn et al., 2015).

The DEA method, whose theoretical foundation traces back to the studies of Farrell (1957) and Charnes, Cooper, and Rhodes (1978), can be applied using two classical models: Constant Returns to Scale (CRS or CCR) and Variable Returns to Scale (VRS or BCC). (Dantas et al., 2016).

The DEA allows for the assessment of relative efficiency through two distinct approaches: input orientation, aimed at minimizing input utilization while maintaining a fixed level of output, and output orientation, in which the goal is to maximize output while keeping inputs at a constant level (Santos & Rover, 2019). An efficiency index equal to 1 indicates that the analyzed DMU is efficient. Both approaches can consider multiple inputs and outputs, generating a ranking of relative efficiency and, thus, providing a range of options for administrative decision-making (Dantas et al., 2016).

DEA models play a crucial role in identifying efficient Decision-Making Units (DMUs) as well as in measuring and pinpointing those considered inefficient. These models utilize a piecewise linear production function to estimate benchmarks for the inefficient DMUs (Costa et al., 2018). The benchmark is determined by projecting the inefficient DMUs onto the efficiency frontier. The model's orientation is determined by how this projection is carried out, with options for input or output orientation. These different approaches provide a valuable perspective for comparative analysis of DMU efficiency, allowing for the identification of improvement opportunities and the direction of efforts towards achieving higher levels of efficiency (Costa et al., 2018).

3.4 Efficiency Model

When analyzing the alternatives within the DEA methodology, the most suitable model for this analysis was the BCC model, as it does not assume proportionality between inputs and outputs. In other words, an increase or decrease in inputs does not necessarily generate the same increase or decrease in outputs (Tschaffon & Meza, 2014).

To calculate the efficiency levels of the DMUs, which included the 26 states of Brazil and the Federal District, Per Capita Expenditure on Public Security was the input used. For the outputs, only Intentional Lethal Violent Crimes (ILVC) were included, defined as deliberate homicides, robbery-homicides, and bodily harm followed by death, as these represent the most severe outcomes and can thus serve as general indicators of violence (Almeida, 2016). It is worth noting that the variables comprising the ILVC indicator were used separately. The ratio of ILVC occurrences per 100,000 inhabitants for each state was used. The inverse of the ILVC rates had to be used because the DEA model interprets that a higher result indicates higher DMU efficiency. When interpreting crime rates, efficiency is associated with a lower ILVC rate, meaning that the variables comprising ILVC are considered undesirable variables, following Lins and Meza (2000).

Although there are other types of significant crimes, ILVC stand out owing to their intent and more severe outcomes, which can directly influence overall violence rates (Almeida & Pitombeira Neto, 2020). Moreover, it is

crucial to maintain a limited set of representative variables in the model, as an excessive number of variables can hinder the DEA from distinguishing the most efficient DMUs (Almeida & Pitombeira Neto, 2020). Table 1 shows the input and output variables chosen for this study.

Table 1 – Variables used in the DEA model

Variables	Description	Calculation	Data source	Reference authors
Input	*Per capita* expenditure on Public Security	Expenses incurred with the Public Security Function, by subfunctions/State population	National Treasury Secretariat /Brazilian Institute of Geography and Statistics	Almeida and Pitombeira Neto (2020); Ribeiro and Longaray (2022), Schull et al. (2014)
Outputs	Intentional Homicide	Intentional Homicide/ 100 thousand inhabitants	Brazilian Public Security Yearbook	Almeida and Pitombeira Neto (2020); Schull , et al. (2014); Ribeiro and Longaray (2022)
	Armed robbery	Robbery / 100 thousand inhabitants	Brazilian Public Security Yearbook	Almeida and Pitombeira Neto (2020); Fernandes (2016); Schull et al. (2014); Ribeiro and Longaray (2022)
	Bodily Injury Followed by Death	Bodily Injury Following Death/ 100 thousand inhabitants	Brazilian Public Security Yearbook	Almeida and Pitombeira Neto (2020); Ribeiro and Longaray (2022)

The model was oriented towards output maximization, which means that minimizing ILVC implies maximizing the inverse of these variables while keeping the inputs constant. This result would not be achievable by reducing inputs. When it comes to public policies, it is more common to attempt improving service delivery with the same resources rather than maintaining service delivery while decreasing resources (Santos & Rover, 2019).

In this manner, based on the model in Table 1, an efficiency scale of public expenditure in Public Security for Brazilian states from 2014 to 2021 was designed using the deaR package as described in Coll-Serrano et al. (2018). Subsequently, the Malmquist Index was calculated to assess the behavior of public expenditure efficiency in public security over the years of the sample. The deaR package, also described in Coll-Serrano et al. (2018), was used for this purpose.

Before obtaining the efficiency scores for each year, a standardized score test (Z-Test)i was conducted to check for outlier observations (Diniz, 2012; Santos & Rover, 2019). Out of 864 observations, 18 were identified as outliers (3 standard deviations above the mean).

It is worth noting that no variable/year combination had more than one outlier; each of the 18 outliers appeared in a distinct variable/year. Additionally, among these 18 outlier observations, 10 were from the state of São Paulo. Upon examining the data specifically for the state of São Paulo (Hair et al., 2009), it was determined that these observations do not deviate from the temporal profile of this Decision-Making Unit (DMU).

Removing a DMU from the study sample, where there is no measurement error, could lead to an overestimation of the new efficiency frontier and eliminate the DMU that acts as a benchmark for the others (Diniz, 2012). Therefore, it was decided to retain the outlier observations in the study.

3.5 Malmquist Index

The Malmquist Index is primarily used to observe the propensity for changes in dynamic efficiency (Malmquist, 1953; Zou et al., 2021). In the current study, it was employed to analyze the intertemporal efficiency of public security in the Brazilian Federal States. The Malmquist Index captures two significant sources of productivity change, gains through technological production changes and through technical efficiency change (Coelli et al., 2005).

Fare et al. (1994) specifies a Malmquist productivity change index based on output as follows:

$$m_0(y_{t+1}, x_{t+1}, y_t, x_t) = \sqrt{\left[\frac{d_0^t(x_{t+1},y_{t+1})}{d_0^t(x_t,y_t)} X \frac{d_0^{t+1}(x_{t+1},y_{t+1})}{d_0^{t+1}(x_t,y_t)}\right]} \quad (1)$$

Where the notation $d^t{}_0(x_{t+1}, yt_{+1})$ represents the distance from the observation of technology from period t+1 to period t. Equation 3 represents the productivity of the production point x_{t+1} and y_{t+1} in relation to the production point x_t and y_t. A value greater than 1 indicates positive productivity growth from period t to period t+1. This index is the geometric mean of two productivity-based Malmquist indices (Coelli et al., 2005). An equivalent way of writing this productivity index is

$$m_0(y_{t+1}, x_{t+1}, y_t, x_t) = \frac{d_0^{t+1}(x_{t+1},y_{t+1})}{d_0^t(x_t,y_t)} \sqrt{\left[\frac{d_0^t(x_{t+1},y_{t+1})}{d_0^{t+1}(x_{t+1},y_{t+1})} X \frac{d_0^t(x_t,y_t)}{d_0^{t+1}(x_t,y_t)}\right]} \quad (2)$$

Where the ratio outside the brackets measures the change in Farrell's output-oriented technical efficiency measure between periods t+1 and t (Coelli et al., 2005). In other words, the variation in efficiency is equivalent to the ratio between the technical efficiency in period t+1 and the technical efficiency in period t. The remaining part of the index in Equation (4) is a measure of technological change. It is the geometric mean of the change in technology between the two periods, valued at x_t and also at x_{t+1} (Coelli et al., 2005). It can be decomposed into two, namely the Technical Efficiency Change Index (IMEf) and the Technological Efficiency Change Index (IMEtec), as follows (Marzzoni , 2022)

$$IMEf = \frac{d_0^{t+1}(x_{t+1},y_{t+1})}{d_0^t(x_t,y_t)} \quad (3)$$

$$IMEtec = \sqrt{\left[\frac{d_0^t(x_{t+1},y_{t+1})}{d_0^{t+1}(x_{t+1},y_{t+1})} X \frac{d_0^t(x_t,y_t)}{d_0^{t+1}(x_t,y_t)}\right]} \quad (4)$$

Just like IM, a value above 1 indicates an increase in IMEf and IMEtec from period t to period t+1. When the value remains at 1, it indicates that there was no change, and when it is less than 1, it indicates a decrease in the indices.

Particularly for IMEtec, the observed change consists in obtaining larger quantities of products without changing the quantities of inputs from one period to the next (Pereira Filho, 2016). The IMEf involves evaluating how close a unit is to the best existing practice within the sample. This implies doing the best possible from one temporal unit to another, considering the available resources and the current technology (Pereira Filho, 2016)

The Malmquist Productivity Index is highlighted as an effective tool for assessing relative efficiency and productivity over time. It allows for comparing the performance of decision-making units across different periods (Zou et al., 2021), which in this study are the Brazilian federative units.

Another advantage is its ability to decompose the total productivity index as presented earlier (Lima & Marinho, 2017). Furthermore, it is noteworthy for its capacity to conduct temporal analyses, identifying productivity changes over distinct periods (Zou et al., 2021).

Therefore, the use of the Malmquist Index in this study serves as a distinguishing factor, contributing to a deeper understanding of productive efficiency and productivity over time.

4 Analysis of Results

In this chapter, we will present the efficiency scales of the allocation of public resources in public security for the Brazilian Federal States from 2014 to 2021, as well as the evolution of this efficiency through Malmquist indices.

4.1 Efficiency Scales

Table 2 shows the efficiency scales for the application of public resources to public security in the Brazilian federative states for the years 2014 to 2021.

Table 2 – Efficiency scales

DMU	2014	2015	2016	2017	2018	2019	2020	2021
Acre	0.54	0.52	0.56	0.90	0.21	0.59	0.37	0.67
Alagoas	0.26	0.49	0.38	0.36	0.63	0.91	0.47	0.82
Amapá	1.00	0.39	0.27	0.26	0.23	0.27	0.32	0.19
Amazon	0.44	0.37	0.34	0.34	0.35	0.36	0.35	0.27
Bahia	0.44	0.54	0.55	0.63	0.59	0.42	0.46	0.45
Ceará	0.62	1.00	1.00	0.99	0.95	1.00	0.57	0.64
Federal District	0.56	0.52	0.57	0.66	0.64	1.00	0.82	0.72
Espírito Santo	0.38	0.70	0.48	0.57	0.58	0.66	0.33	0.32
Goiás	0.28	0.31	0.28	0.33	0.31	0.44	0.48	0.49
Maranhão	0.64	0.56	1.00	0.89	0.86	0.98	0.85	1.00
Mato Grosso	0.31	0.37	0.34	0.39	0.42	0.34	0.38	0.46
Mato Grosso do Sul	0.47	0.54	0.46	0.66	0.58	0.54	0.49	0.54
Minas Gerais	1.00	1.00	1.00	1.00	1.00	1.00	0.81	0.82
Pará	0.31	0.36	0.28	0.26	0.62	0.40	0.32	0.36
Paraíba	1.00	0.66	0.90	0.77	0.61	0.74	0.65	0.73
Paraná	0.65	0.67	0.62	0.92	0.60	0.48	0.56	0.74
Pernambuco	0.58	0.62	0.45	0.28	1.00	0.67	0.88	0.75
Piauí	1.00	1.00	0.74	1.00	1.00	0.62	1.00	1.00
Rio de Janeiro	0.47	0.83	0.43	0.50	0.56	0.67	0.66	0.50
Rio Grande do Norte	0.34	0.44	0.51	0.33	0.27	0.25	0.20	0.25
Rio Grande do Sul	0.52	0.58	0.48	0.54	0.69	0.71	0.58	0.56
Rondônia	0.47	0.77	0.35	0.58	0.58	0.34	1.00	0.48
Roraima	0.87	0.54	0.39	0.40	0.18	0.22	0.25	0.20
Santa Catarina	0.94	0.76	0.80	0.69	0.80	1.00	1.00	1.00
São Paulo	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Sergipe	0.66	0.31	1.00	0.50	0.63	0.76	0.94	0.45
Tocantins	0.57	1.00	0.63	0.69	0.47	0.35	0.32	1.00
Average	0.60	0.62	0.59	0.61	0.61	0.62	0.59	0.61
Standard deviation	0.25	0.23	0.25	0.25	0.25	0.27	0.26	0.26

Table 2 reveals that São Paulo stands out as the sole consistently efficient Decision-Making Unit (DMU) across all analyzed years, serving as a benchmark in public security resource allocation. Fernandes (2021) demonstrates that São Paulo had a homicide rate of 43.2 cases per 100,000 inhabitants in 1999. This number dropped to 22 cases per 100,000 inhabitants by 2005 (Fernandes, 2021). In the year 2021, this figure was 6.64 cases of ILVC per 100,000 inhabitants according to ABSP data from 2022. It can be argued that the reconfiguration of the policing system, stemming from initiatives aimed at harmonizing police jurisdictions and implementing the INFOCRIM system, played a significant role in reducing homicide rates in São Paulo (Fernandes, 2021).

Minas Gerais is efficient from 2014 to 2019 but loses its benchmark status in 2020 and 2021. Notably, Ribeiro and Longaray (2022) and Almeida and Pitombeira Neto (2020) also identified São Paulo's efficiency in 2020. Marzzoni (2022) found São Paulo and Santa Catarina on the efficiency frontier from 2014 to 2020, with Santa Catarina also in 2019 and 2020.

The states that appeared as the most inefficient in public security resource allocation were Rio Grande do Norte, Amazonas, Pará, and Goiás. Recently, Mossoró, a municipality in Rio Grande do Norte, was ranked as the 11th most violent city in the world, and the most violent in Brazil, according to a study by *Seguridad, Justicia y Paz* (SJP, 2022).

From 2014 to 2021, the annual average efficiency in public security resource allocation held steady at around 0.60, with a standard deviation of 0.25, implying room for improvement without increased funding. While Monte and Leopoldino (2020) found a 2017 average efficiency of 0.65, Freitas Júnior et al. (2020) reported a higher 0.69 for 2011 to 2015. However, this study reveals a decline in efficiency, highlighting challenges and emphasizing the urgency of strategic improvements in Brazilian states' public security resource allocation.

4.2 Malmquist Index Analysis

Table 3 shows the results found by calculating the MI, which is the outcome of the product between the index of technical efficiency change and the index of technological efficiency change. The MI assesses the change in efficiency from one year to another.

Table 3 – Malmquist Index *

DMU	2014 - 2015	2015 - 2016	2016 - 2017	2017 - 2018	2018 - 2019	2019 - 2020	2020 - 2021
Acre	0.74	1.49	0.84	0.23	2.87	1.41	1.68
Alagoas	1.89	0.86	0.99	1.54	1.80	0.60	1.70
Amapá	0.07	0.77	1.06	1.20	1.08	1.20	0.54
Amazon	0.66	0.86	1.22	1.13	1.06	1.13	0.93
Bahia	0.90	0.89	1.22	1.15	0.93	1.26	1.03
Ceará	1.10	0.71	0.89	1.41	1.36	0.73	1.10
Federal District	0.54	1.20	0.99	0.75	1.80	0.82	0.94
Espírito Santo	1.34	0.65	1.20	1.06	1.41	0.61	0.91
Goiás	0.82	0.67	1.54	0.97	1.68	1.66	1.02
Maranhão	0.63	0.87	1.03	1.43	1.02	0.99	1.31
Mato Grosso	0.71	0.75	1.25	1.36	0.77	1.35	1.29
Mato Grosso do Sul	1.21	0.75	1.73	0.87	1.17	1.14	0.92
Minas Gerais	0.40	1.56	1.09	1.25	1.14	1.12	1.00
Pará	0.87	0.60	0.96	1.47	0.70	0.89	1.18
Paraíba	0.56	1.00	0.84	0.75	1.28	1.03	1.13
Paraná	0.69	0.88	1.55	0.80	0.91	1.59	1.32
Pernambuco	1.12	0.64	0.63	2.24	0.77	1.35	0.91
Piauí	0.25	0.31	1.07	1.09	0.67	1.62	1.07
Rio de Janeiro	1.09	0.53	1.09	0.97	1.40	1.46	0.72
Rio Grande do Norte	1.00	0.94	0.70	1.00	1.07	1.04	1.30
Rio Grande do Sul	1.27	0.93	1.15	1.12	1.12	0.90	0.95
Rondônia	1.37	0.65	1.61	1.29	0.60	5.70	0.32
Roraima	0.45	0.78	1.06	0.64	1.55	0.89	0.69
Santa Catarina	0.88	1.06	0.92	1.46	1.61	1.24	1.00
São Paulo	0.54	1.19	0.83	0.89	1.08	1.00	1.03
Sergipe	0.13	6.30	0.21	0.89	1.40	1.90	0.30
Tocantins	1.47	0.57	1.08	0.70	0.66	1.09	5.87
Productivity increase	9.00	6.00	16.00	15.00	19.00	18.00	14.00
Decrease in productivity	17.00	20.00	11.00	11.00	8.00	8.00	12.00
MI of the total sample	0.70	0.87	1.00	1.00	1.14	1.18	1.00

* When the MI is less than 1, it means that there was a reduction in DMU productivity; when equal to 1, the productivity is maintained; and when greater than 1, there is an increase in productivity from one year to the next (Zou et al., 2021)

There is a higher number of DMUs with increased efficiency from 2018 to 2019, with 19 states showing an increase in the MI during this period. However, there is a decrease in productivity from 2019 to 2020, with 18 DMUs having an MI greater than 1, and from 2020 to 2021, only 14 DMUs have an MI greater than 1.

Notable is the MI of the state of Sergipe in the period from 2015 to 2016, which is 6.30. Table 2 shows that this DMU ranges from an efficiency index of 0.31 in 2015 to an efficiency index of 1 in 2016. Similarly, the state of Tocantins, whose MI was 5.87 in the period from 2020 to 2021, showed an efficiency index of 0.32 in 2020 and is on the efficiency frontier in 2021.

In 2014-2015, MI reflects a 30% productivity decline (MI=0.70), and in 2015-2016, it indicates a 13% drop (MI=0.87), suggesting reduced efficiency in public security resource allocation for Brazilian states. Despite the Brazilian Public Security Yearbook reporting a 2% ILVC decrease from 2014 to 2015, it wasn't sufficient to improve sector performance given per capita expenditure. The subsequent 4% ILVC increase from 2015 to 2016 possibly contributed to decreased productivity, as indicated by MI.

In the periods from 2016-2017, 2017-2018, and 2020-2021, the MI is equal to 1, MI indicates constant productivity in public security resource allocation. According to BPSY, there was a 2.9% increase in ILVC between 2016-2017, a reduction of 10.8% between 2017-2018, and a 6.5% reduction between 2020-2021. However, in terms of per capita expenditure on public security, the performance of this public administration function remained constant.

In 2018-2019 and 2019-2020, the MI is greater than 1, indicating there was an enhancement in the efficiency of public security resource allocation for Brazilian states in general. There was a 19.31% reduction in ILVC from 2018-2019, justifying the improved efficiency indicated by the MI, which reflects a 14% increase in productivity during this period. Even with a 4% increase in ILVC from 2019 to 2020, this period showed a more substantial increase in productivity of 18%. This result can be attributed to the level of public resource allocation in this sector.

4.3 Technical Efficiency Change Index and Technological Efficiency Change Index

Tables 4 and 5 show the results found for IMEf and IMEtec respectively, as presented in Section 3.

Table 4 – Technical Efficiency Change Index

DMU	2014 - 2015	2015 - 2016	2016 - 2017	2017 - 2018	2018 - 2019	2019 - 2020	2020 - 2021
Acre	2.81	1.76	1.35	0.26	2.33	1.28	1.76
Alagoas	5.71	0.95	0.96	1.38	1.74	0.56	1.70
Amapá	0.20	0.95	1.03	0.96	0.79	1.00	0.56
Amazon	3.29	1.49	1.19	0.91	0.79	1.10	0.90
Bahia	4.63	1.68	1.18	0.92	0.69	1.11	1.00
Ceará	5.67	1.35	0.87	1.12	1.00	0.62	1.07
Federal District	1.63	1.28	1.11	0.92	1.73	0.79	0.89
Espírito Santo	6.93	1.24	1.16	0.96	1.17	0.58	0.86
Goiás	2.48	1.08	1.50	0.78	1.23	1.52	0.97
Maranhão	3.05	1.66	1.00	1.38	0.94	0.92	1.37
Mato Grosso	3.65	1.41	1.21	1.09	0.56	1.20	1.28
Mato Grosso do Sul	3.65	1.00	1.68	0.79	1.00	1.09	0.90
Minas Gerais	2.09	2.96	1.06	1.00	0.83	0.97	0.97
Pará	2.63	0.81	0.93	2.11	0.67	0.84	1.12
Paraíba	1.81	1.62	0.82	0.81	1.16	0.96	1.11
Paraná	3.59	1.66	1.50	0.64	0.71	1.43	1.31
Pernambuco	4.43	1.00	0.61	3.51	0.69	1.14	0.97
Piauí	1.00	0.60	1.04	1.27	0.68	1.59	1.08
Rio de Janeiro	3.58	1.00	1.05	1.03	1.29	1.37	0.69
Rio Grande do Norte	5.24	1.77	0.68	0.81	0.79	0.92	1.29
Rio Grande do Sul	4.78	1.42	1.12	0.93	1.05	0.85	0.90
Rondônia	5.77	1.05	1.56	1.27	0.44	4.88	0.35
Roraima	1.72	1.30	1.03	0.51	1.30	0.92	0.64
Santa Catarina	2.68	1.89	0.89	1.17	1.21	1.10	1.00
São Paulo	1.64	1.05	1.00	1.00	1.00	1.00	1.00
Sergipe	0.53	7.00	0.35	1.24	1.28	1.59	0.31
Tocantins	7.59	1.08	1.05	0.56	0.53	1.04	5.97
Productivity increase	24.00	20.00	17.00	11.00	11.00	14.00	11.00
Decrease in productivity	2.00	4.00	8.00	16.00	13.00	12.00	13.00
IMEf of the entire sample	2.76	1.37	1.03	0.98	0.95	1.08	1.00

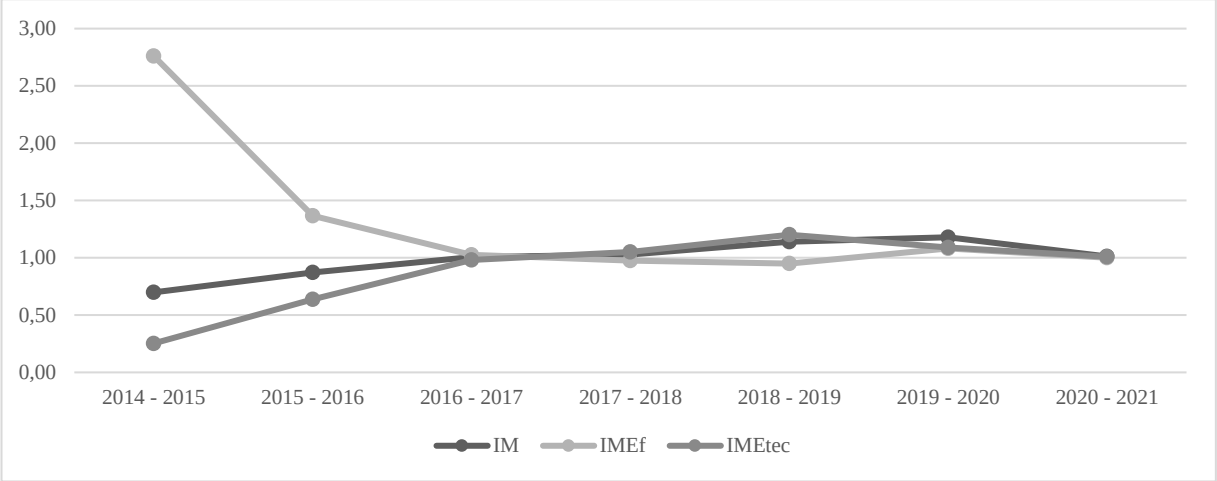
Table 5 – Technological Efficiency Change Index

DMU	2014 - 2015	2015 - 2016	2016 - 2017	2017 - 2018	2018 - 2019	2019 - 2020	2020 - 2021
Acre	0.26	0.85	1.61	0.89	1.23	1.10	0.96
Alagoas	0.33	0.90	0.93	1.12	1.04	1.07	1.00
Amapá	0.33	0.81	0.96	1.25	1.36	1.18	0.96
Amazon	0.20	0.58	1.01	1.25	1.35	1.03	1.04
Bahia	0.19	0.53	1.15	1.25	1.35	1.14	1.03
Ceará	0.19	0.53	0.99	1.25	1.36	1.19	1.02
Federal District	0.33	0.94	1.16	0.82	1.04	1.04	1.06
Espírito Santo	0.19	0.53	1.18	1.10	1.20	1.05	1.06
Goiás	0.33	0.63	1.21	1.25	1.36	1.09	1.05
Maranhão	0.21	0.53	0.89	1.04	1.08	1.08	0.96
Mato Grosso	0.20	0.53	1.13	1.25	1.36	1.13	1.01
Mato Grosso do Sul	0.33	0.73	1.41	1.10	1.16	1.05	1.03
Minas Gerais	0.19	0.53	1.00	1.25	1.36	1.15	1.03
Pará	0.33	0.74	0.92	0.69	1.05	1.05	1.05
Paraíba	0.31	0.63	0.85	0.93	1.11	1.08	1.02
Paraná	0.19	0.53	1.48	1.25	1.28	1.11	1.00
Pernambuco	0.25	0.62	0.63	0.64	1.11	1.18	0.94
Piauí	0.25	0.53	1.34	0.86	0.99	1.02	0.99
Rio de Janeiro	0.30	0.53	1.15	0.95	1.09	1.07	1.04
Rio Grande do Norte	0.19	0.53	0.66	1.25	1.36	1.13	1.00
Rio Grande do Sul	0.27	0.65	1.12	1.20	1.06	1.05	1.06
Rondônia	0.24	0.62	1.65	1.02	1.36	1.17	0.93
Roraima	0.26	0.60	1.02	1.25	1.19	0.96	1.08
Santa Catarina	0.33	0.56	0.86	1.25	1.33	1.13	1.00
São Paulo	0.33	1.13	1.00	0.89	1.08	1.01	1.03
Sergipe	0.25	0.90	0.50	0.72	1.09	1.20	0.96
Tocantins	0.19	0.53	1.09	1.25	1.25	1.04	0.98
Productivity increase	0.00	1.00	15.00	18.00	26.00	26.00	19.00
Decrease in productivity	27.00	26.00	10.00	9.00	1.00	1.00	8.00
IMEtec of the entire sample	0.25	0.64	0.98	1.05	1.20	1.09	1.01

Between 2014-2015, 24 DMUs experienced an increase in IMEf, with only 2 DMUs showing a decrease. In contrast, from 2020-2021, only 11 DMUs improved IMEf, while 13 DMUs declined. When examining IMEtec, all DMUs exhibited a reduction in this indicator from 2014 to 2015, and only one DMU demonstrated an increase in technological productivity in the period from 2015 to 2016 (São Paulo). However, from 2016 to 2017, there was a considerable increase in DMUs with improved technological productivity, according to IMEtec, with a growing number of DMUs experiencing technological change enhancement in each subsequent period, reaching its peak in the last period from 2020 to 2021.

In the initial years under study, states that achieved an increase in IM did so primarily owing to the influence of IMEf. This trend reverses from the period of 2016 to 2017 onward, i.e., municipalities that achieved increased productivity according to IM did so with a greater influence of IMEtec. Figure 2 shows the evolution of IM, IMEf, and IMEtec for the entire study sample.

Figure 2 - Malmquist productivity indices



Malmquist Index; IMEf: Technical Efficiency Change Index; IMEtec: Technological Efficiency Change Index

One can observe that the MI of the total sample grows over the periods until the last period of 2020-2021, when there was a decline in the overall MI. Meanwhile, the IMEf of the total sample showed a decline over the years with a slight increase in the 2019-2020 period. Finally, the IMEtec increased until the 2018-2019 period and started decreasing in the last two study periods. In his study, Pereira Filho (2016) reported an almost exclusive contribution of the technological change index to the increase in efficiency in public security. According to the author, this essentially reflects changes in the position of the efficiency frontier, indicating innovations and progress in production technology that contributed to the increase in the production of public goods (Pereira Filho, 2016).

4.4 Discussion

The rate of lethal violence (ILVC) reached its peak in 2017, when the country reported a rate of 30.9 ILVC per 100,000 inhabitants. Starting in 2018, a trend of declining mortality rates was established (Lima et al., 2022). In 2021, Brazil recorded a ILVC rate of 22.3 per 100,000 inhabitants, representing a 6.5% reduction compared to 2020 (Lima et al., 2022).

This result is corroborated in the Malmquist Index (IM) in Table 3. There was a decrease in the IM from 2014 to 2016, the performance remained constant from 2016 to 2018, and there was a productivity gain from 2018 to 2020. The IM remained constant from 2020 to 2021. Even with a 6.5% decrease in the ILVC rate during this period, there may have been a reduction in investment in public security by Brazilian states, which explains the maintenance of productivity during this period.

However, this reduction is not uniform; Amapá, Bahia, Amazonas, Ceará, and Roraima saw an increase in ILVC from 2020-2021 (Lima et al., 2022). These five states had the eight worst efficiencies found by the study for this period (0.1859, 0.1952, 0.2660, and 0.4459, respectively), along with Rio Grande do Norte, Espírito Santo, and Pará (0.2526, 0.3159, and 0.3566).

The literature provides several hypotheses to explain the declining trend in ILVC rates. Lima et al. (2022) suggested four of them: (1) demographic changes; (2) violence prevention policies, and integrated policing models; (3) the action of organized crime; and (4) gun control and recent changes in legislation. It is not the objective of the current study to identify the factors that influence the behavior of public resource efficiency in public security by the Federative Units of Brazil. However, the results of this study are aligned with findings from the literature regarding the average efficiency in the allocation of public resources in public security by Brazilian states (Almeida & Pitombeira Neto, 2020; Marzzoni, 2022; Ribeiro & Longaray, 2022, Freitas Júnior et al., 2020; Monte & Leopoldino, 2020; Pereira Filho, 2016), as well as the Technological Change Index (Pereira Filho, 2016). The present findings are also in line with the trends presented by the Brazilian Public Security Yearbook of 2022, regarding the 2021 data.

Concerning demographic changes, Lima et al. (2022) highlighted that demographic change plays a significant role, contributing to a 23% reduction in the homicide rate in the country between 2004 and 2020. Most of the reduction in the overall homicide mortality rate (77%) resulted from a decreased mortality risk among various population groups. In this sense, it is essential for the government to develop violence prevention policies tailored

to the risk profiles of each age group, rather than relying solely on reactive and repressive approaches in public security.

Kopittke and Ramos (2021) underscored that the country has innovative programs that have demonstrated to be effective in reducing homicides and violence, such as outcome-focused management programs, restrictions on the sale of alcoholic beverages, the implementation of Municipal Guards, the adoption of the Disarmament Statute, and measures provided in the Maria da Penha Law. For example, programs such as the Integration Program of Public Security Management (IGESP) in Minas Gerais, Infocrim in São Paulo, and the State Present Program in Espírito Santo produced a drop in homicide rates ranging from 9% to 17% (Kopittke & Ramos, 2021). Although Espírito Santo did not demonstrate efficiency in any of the study years, there was an increase in productivity between the periods of 2014 to 2015 and 2016 to 2019 (Table 3). On the other hand, the state of Minas Gerais demonstrated efficiency for almost all years in the study period, except for 2020 and 2021 (Table 2), and showed an increase in productivity for the periods from 2015 to 2020 (Table 3). The state of São Paulo was the only one that remained on the efficiency frontier for all years in the study period (Table 2). Kopittke and Ramos (2021) argued that although Brazil adopted the Evidence-Based Public Security paradigm much later, when compared to international development, this approach has considerable potential to improve public spending efficiency, reduce violence, and overcome decision-making approaches anchored in beliefs, corporate interests, biases, and populist political considerations. The present study provides evidence that supports the arguments of these authors.

Research has already shown that fluctuations in lethal violence rates have been widely affected by the dynamics of criminal markets in Brazil and the actions of criminal groups (Lima et al., 2022; Manso & Dias, 2018; Oliveira, 2022). The considerable increase in homicides in 2017, as mentioned earlier, was likely caused by the rivalry between drug trafficking factions and the crisis in the penitentiary system (Lima et al., 2022; Oliveira, 2022). Starting in 2018, these regional conflicts between criminal groups began to calm down. This was partly due to the significant expansion and establishment of territories by the Red Command (Comando Vermelho) in the northern region, especially in the states of Pará and Acre (Lima et al., 2022). On the other hand, in some areas, such as Amazonas, conflicts were intensified after a period of relative stability (Lima et al., 2022). It is no coincidence that the state of Amazonas presented the second-worst average efficiency over the study years in this work (0.3525), after the state of Rio Grande do Norte (0.3237). Although these conflicts may have diminished, the drop in lethal violence in 24 states of the country suggests that other variables should be examined (Oliveira, 2022). Authors such as Bohn et al. (2015) and Freitas Júnior (2020) reported factors that can influence the efficiency of public resource allocation in public security, such as population density, quality of education, level of social inequality, unemployment rate, and police force effectiveness, among others. However, these factors are not exhaustive, and the Brazilian literature lacks enough research for a deeper understanding of the elements influencing the efficiency of public resource allocation in public security, as well as their relationships with factors that influence the increase and/or decrease in crime itself.

The international literature and existing scientific research relate the increase in the number of firearms in circulation to an increase in lethal violence and other crimes (Duggan, 2001; Peres et al., 2011). This indicates that individuals who have access to firearms generally opt for more violent responses in resolving interpersonal conflicts (Lima et al., 2022). Given the policy of relaxed regulations, oversight, and tracking of firearms and ammunition implemented by the previous government (2019-2022), it is essential to emphasize that the available data do not support the claim that the increase in the number of firearms in circulation, especially through licenses granted to Hunters, Shooters, and Collectors (CAC), has led to a reduction in intentional violent death rates in Brazil (Lima et al., 2022). According to the Brazilian Public Security Forum of 2022, there was a 473.6% increase in active CAC registrations in Brazil (BPSY, 2022).

Another explanation could be attributed to the COVID-19 pandemic. As an additional test, we analyzed the pre-COVID-19 pandemic period (2014-2018) and the COVID-19 pandemic period (2019-2021). We conducted the Kruskal-Wallis test for non-parametric data considering the pre-pandemic period (mean=1.05512) and pandemic period (mean=1.2574), yielding a p-value of 0.08412. At a significance level of 10%, it can be stated that during the COVID-19 pandemic period, Brazilian states exhibited a higher average improvement in the efficiency of public resource allocation towards public security. In other words, there are indications that the efficiency of public resource allocation for public security improved during the COVID-19 pandemic. Two reasons may have driven this improvement: (i) the decrease in ILVC due to isolation measures (Lima et al., 2022); (ii) even if there was no decrease in ILVC rates, there was a reduction in public safety expenditures during this period as public budgets focused on addressing the health crisis (Peres & Bueno, 2022).

However, these results should be analyzed with caution and should not be generalized. It is suggested to study more deeply the effects of the COVID-19 pandemic on the efficiency of public resource allocation towards public security.

The analysis of public spending efficiency on public security in Brazilian states reveals potential improvement, indicated by an average efficiency of 0.60 and a standard deviation of 0.25. Similar conclusions from other studies (e.g Freitas Júnior, 2020; Monte & Leopoldino, 2020; Pereira Filho, 2016) suggest the opportunity to enhance efficiency in public security spending by Brazilian states. Pereira Filho's (2016) twelve-year assessment (2000 to 2011) found an average efficiency of 0.668, contrasting with this study ending ten years later, showing a decreased average efficiency. This underscores persistent deficiency in Brazilian states' public security performance from 2014 to 2021, aligned with Pereira Filho's findings. Comparably, Freitas Júnior et al. (2020) found a 2011-2015 average efficiency of 0.686, indicating consistency with the current study's results and supporting the absence of a difference in average efficiency of public resource allocation in public security.

In this way, the study achieves its objective of analyzing the efficiency of public resource allocation towards public security in Brazilian states and examining how this efficiency behaves over time.

5 Conclusions

The goal of this work was to analyze the efficiency of the application of public resources in the public safety of the Brazilian Federative States and the behavior of this efficiency over time. The work achieves its goal by developing a DEA model and applying the Malmquist Productivity Index

The study has limitations in the choice of variables for the DEA model used to measure the efficiency of public resource allocation in public security from 2014 to 2021. Designing DEA models with different variables may yield different results; however, there is a trend when comparing the results of this study to previous studies; for example, São Paulo is considered efficient for all years in the sample (Almeida & Pitombeira Neto, 2020; Marzzoni, 2022; Pereira Filho, 2016). Furthermore, the results presented in the study should not be generalized.

The study contributes to the literature by analyzing the behavior of the efficiency of public resource allocation in public security and providing insights into how this efficiency was influenced over time by changes in technical and technological productivity. It also contributes by employing a methodology to analyze the behavior of the efficiency of public resource allocation in public security over time, as in the application of the Malmquist productivity index.

Another significant contribution of this study is its ability to provide information that can serve as a basis for decision-making by authorities responsible for public security, as they can rely on those states that were considered efficient in the proposed model. Identifying areas with potential for improvement in efficiency can result in more effective policies and more efficient allocation of resources. This is particularly relevant to public interest, given that the focus of the study is a fundamental sector for the quality of life of the population.

Future research should make a more in-depth analysis of each analyzed period to clarify the reasons for the decrease and increase in technical and technological productivity and, consequently, in the Malmquist Index. Another suggestion is to use other DEA models - to determine how the efficiency of public resource allocation in public security behaves - as well as other DEA methodologies such as Dynamic Network DEA (Tone & Tsutsui, 2014). One last suggestion is to investigate exogenous variables to understand the explanatory factors for the efficiency of public resource allocation in public security and its behavior over time. It is known that factors such as education, social inequality, and unemployment affect public security and, consequently, can influence the efficiency of public resource allocation in public security.

It is also emphasized as a suggestion to investigate how political cycles influence the allocation of public resources in public safety and to conduct more in-depth studies on the effect of the COVID-19 pandemic on the efficiency of this state function.

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[i] $Z_{it} = \frac{x_{it} - \bar{X}}{S}$, Where, $\bar{X}$ is the sample mean, S is the sample standard deviation, and x_{it} is observation i in year t.