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ARTICLE

# Internship, research and geometry in early childhood education: a study on (de)composition of geometric figures

## *Estágio, pesquisa e geometria na educação infantil: um estudo sobre (de)composição de figuras geométricas*

## *Prácticas, investigación y geometría en la educación infantil: un estudio sobre (des)composición de figuras geométricas*

Simone Damm Zogaib[2]
Vânia Maria Pereira dos Santos-Wagner[3]



**Abstract:** The text that shows the relation between internship, research and geometry in early childhood education. It results from research on (de)composition of geometric figures with a kindergarten class in semi-boarding school, during the internship program of a pedagogy course of Sergipe public university. As theoretical support we have Clements and Sarama (2011), Lorenzato (2010), Mendes and Delgado (2008), Pimenta and Lima (2012), Horn (2016), Smole (2014) and Vygotsky (2001). We emphasize that children possess knowledge of geometric figures, although restricted to experienced visual prototypes. Such knowledge can be problematized with activities of (de)composition of figures for the development of spatial skills, but there is a need to deepen geometry's teacher knowledge. We conclude that the articulation of the internship/research/geometry contributes to the initial teacher education of nursery and kindergarten schoolteachers while enabling theoretical and practical knowledge in the field of geometry for children.

**Keywords:** Early child education. Relationship internship and research. (De)composition of geometric figures.

***Resumo:*** *O texto apresenta relação entre estágio, pesquisa e geometria na educação infantil. Resulta de pesquisa qualitativa sobre (de)composição de figuras geométricas com turma de educação infantil em semi-internato, durante estágio de um curso de pedagogia de universidade pública sergipana. Como suporte teórico temos: Clements e Sarama (2011), Lorenzato (2010), Mendes e Delgado (2008), Pimenta e Lima (2012), Horn (2016), Smole (2014) e Vygotsky (2001). Ressaltamos que crianças possuem conhecimentos de figuras geométricas, ainda que restritos aos protótipos visuais vivenciados. Tais conhecimentos podem ser problematizados com atividades de (de)composição de figuras para desenvolvimento de habilidades espaciais, mas há necessidade de aprofundamento de conhecimento docente de geometria. Concluímos que a articulação estágio/pesquisa/geometria contribui para formação inicial de professores de educação infantil ao possibilitar conhecimentos teóricos e práticos no campo da geometria para crianças.*

***Palavras-chave:*** *Educação infantil. Relação estágio e pesquisa. (De)composição de figuras geométricas.*

***Resumen:*** *El texto que muestra la relación entre prácticas, investigación y geometría en la educación infantil. Se trata de una investigación sobre (des)composición de figuras geométricas en una clase de educación infantil en semi-internado, durante la prácticas de un curso de pedagogía de la universidad pública sergipana. Como soporte teórico tenemos: Clements y Sarama (2011), Lorenzato (2010), Mendes y Delgado (2008), Pimenta y Lima (2012), Horn (2016), Smole (2014) y Vygotsky (2001). Resaltamos que los niños tienen conocimientos de figuras geométricas, aunque restringidos a los prototipos visuales establecidos. Estos conocimientos pueden ser problematizes con activities de (des) composición de figuras para el desarrollo de habilidades espaciales, siendo necesario profundizar en el conocimiento docente de geometria. Llegamos a la conclusión de que la articulación prácticas/ investigación/geometría contribuye a la formación inicial de profesores de educación infantil, y posibilita los conocimientos teóricos y prácticos en el campo de la geometría para niños.*

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[2] Federal University of Sergipe (UFS) – Email: simonedammzogaib@gmail.com
[3] Federal University of Rio de Janeiro (UFRJ) and Federal University of Espírito Santo (UFES) - Email: profvaniasantoswagner@gmail.com

***Palabras clave:*** *Educación infantil. Relación con las prácticas y investigación. (Des)composición de figuras geométricas.*

## Introduction

The purpose of this text is to present reflections about the relationship between teacher training, research and geometry in early childhood education and possible contributions to the training of future teachers of this stage of basic education. It is based on research carried out with children together with students of the discipline of Supervised Teacher Training in Early Childhood Education of the Federal University of Sergipe (UFS). The conception of traineeship as a research-oriented practice for teacher training involved: a) analysis of contexts where the trainings were completed; b) development of the trainees in relation to the postures and own abilities of researchers; c) elaboration of projects that make it possible to understand and problematise situations observed and experienced; and d) search for a knowledge that relates existing explanations and new data produced in the field of research and training (PIMENTA; LIMA, 2012).



This proposal ranged from a certain "training culture" based on a standard work model for trainees in school institutions, namely: observation, registration, regency and reporting. Such model consisted of planning a series of activities, usually in the form of a project, and applying what was planned in the school-field internship. It was (and still is) common to come up with a series of mostly "fantastic" or "fanciful" classes, filled with diversified and colorful resources, often far removed from the everyday classroom and children. And, after that period of "news", the school returned to its "normality". This way of thinking about the internship bothered us and what we proposed to the students of Pedagogy was in line with the ideas of Pimenta and Lima (2012), as we have already mentioned, and of Horn (2016, p. 37), who conceives the supervised training as:

> a power that triggers in the students of the Pedagogy Course, on the eve of the completion of their undergraduate courses, a strenght to think the education and the teaching practice in other ways in Early Education schools. Thus, an investigative practice, permeated with studies carried out along the different supervised trainings, which contributes to the training of educational professionals capable of investigating their pedagogical actions and their experiences with the schools (our translation).

Thus, our intention was to rethink the trainings seen as a bureaucratic function, or as an imitation of models, and to open paths for future teachers to reflect on a school that is wanted and needed, within the existing conditions. There were 17 (seventeen) camp-field schools, of which 12 (twelve) were from the public-school system and 05 (five) from the private network. The 46 (forty-six) students of the discipline were divided into groups for the accomplishment of the internship in this perspective, articulated with the research. After reading and discussing with the graduates about this conception, the work was organized as follows: participant observation of nursery schools; record of observations in field journals; problematization of this first experience; elaboration and execution of an investigative

proposal that presented a research problem, theoretical and methodological reference and tasks for children in kindergarten.

After two weeks of participant observation, the students returned to the university with their records in field journals. They were rich documents of experiences with children, teachers, managers and other actors in the schools of early childhood education. By focusing on those registers, the licensees identified a situation and/or knowledge gap that they have problematised and for which they have developed a research proposal. They devoted themselves to a bibliographic study related to the research topic. There were 17 research proposals grouped into three themes: a) playfulness and movement in early childhood education (06 works); b) adventures in the magical world of reading and children's literature (09 works); and c) the teaching of mathematics in early childhood education (03 works).

For this article we have selected one of the research works related to "mathematics teaching in children's education", specifically in the field of geometry in childhood. Three reasons led us to select it. The first one refers to the surprise by the choice of the theme by a group of students. In our experiences as teacher trainers, there was little work concerning mathematics for preschool children. The second reason stems from the scarcity of literature on the subject. Finally, we underline the importance of an intentional work with mathematics in early childhood education and, consequently, the need to think about teacher training in this context.



For this group, we proposed a qualitative study of the exploratory and descriptive type (FIORENTINI; LORENZATO, 2007; LÜDKE; ANDRÉ, 2013), using a teaching experiment (ROMBERG, 1992; SILVA; SANTOS-WAGNER, 2009; STEFFE; THOMPSON, 2000) with an emphasis on geometry for children (LORENZATO, 2010; SMOLE, 2003). We emphasise that an educational experiment, according to these authors, consists of a research methodology used in mathematical education and that is in line with the conception of internship and research already mentioned. In addition, a teaching experiment involves the elaboration and application of a set of tasks by researchers and/or teachers in the classroom at various times called teaching episodes. These episodes are tracked, recorded, reworked and analysed by the researchers and/or teachers involved in the research process.

At the time, the authors studied by the group were Smole (2003; 2014), Smole, Diniz and Candido (2003) and Lorenzato (2010). From this theoretical support, and mainly based on the activities proposed in the work of Lorenzato (2010), a teaching experiment was designed with 12 activities involving space skills. These tasks were performed in a semi-boarding school in Aracaju-SE, which provided care for about 50 children from disadvantaged classes in the community. The teaching experiment took place in a kindergarten class with 18 children between 4 and 5 years old for two months. After applying the proposed tasks, Pedagogy students presented their oral and written reports, socialising their internship/research experiences.

In view of the rare Brazilian publications on geometry in early childhood education, we consider it important to analyse in depth the data produced in that teaching experiment. Regarding the scarcity of works, we carried out a mapping of the academic productions in the southeastern Brazilian region. In summary, we have: between 2005 and 2015, of the 44

public and private universities surveyed we have found that, only 06 universities presented studies related to mathematics in early childhood education. They are: Pontifical Catholic University of São Paulo (PUC-SP), Pontifical Catholic University of Campinas (PUC-Campinas), State University of Campinas (UNICAMP), State University of São Paulo (UNESP), São Francisco University (USF) and University of São Paulo (USP). There are 11 papers, being 09 master's dissertations and 02 doctoral theses. Regarding the geometry in child education, until the moment, we find a production referring to continuous teacher education.

The mapping reinforced our intention to analyse the findings of this research articulated to the stage in early childhood education. It has led us to reflect mainly on how to work mathematics with young children from what they say and do. When children arrive at kindergarten, they already carry within themselves what Steffe and Thompson (2000) call "students' mathematics". This refers to a mathematics of students or children, usually distinct from ours. It is indicated by what they say and do when they engage in daily activities in and out of school. According to these authors a "looking behind" is essential, that is, looking behind what children say and do to try to understand and work with their mathematical realities.



In this context, we have a dialogue with the official and national documents related to children's education, with studies of national research already done and with international studies that we find on this subject. Given the large amount of data produced, we chose to show here one of the activities of composition and decomposition of figures. This task was carried out by a group of 04 children from a semi-boarding pre-school Sergipe class: Sophia, Mateus, Ricardo and Samuel (fictitious names). The choice of this activity was due to the following reasons: a) possibility of relating the spatial ability of composition and decomposition with knowledge about geometric figures, since the children, students of Pedagogy and the teacher of the group knew squares, rectangles, circles and triangles; b) importance that the work with this ability has for the mathematical/geometric thinking of children, for their life in the present and in the future; and c) the opportunity to problematise this mathematical/geometric thinking of children in early childhood education and relate it to the training of future teachers of this stage of basic education.

Thus, we intend in this article to reflect on a question: What contributions this research articulated to the training brings to the education of early childhood school teachers, and especially to the field of geometry in childhood? To that end, we started the text by addressing some theoretical assumptions regarding children, early childhood education, and initial teacher training with emphasis on the articulation between training, research and geometry in childhood. Next, we analysed one of the tasks of composition and decomposition of figures, performed by a kindergarten class of the semi-boarding school in Sergipe. We, then, discussed aspects related to the geometric knowledge involved in the task, and a problematisation of this knowledge articulated to the initial formation of teachers of early childhood education.

## THEORETICAL ASSUMPTIONS AND THE RELATION TEACHER TRAINING, RESEARCH AND GEOMETRY IN CHILD EDUCATION

ARTICLE

# Theoretical assumptions and the relation teacher training, research and geometry in child education

## *Thinking Children, Early Childhood Education and Teacher Training*

According to the National Curriculum Guidelines for Early Childhood Education [Diretrizes Curriculares Nacionais para a Educação infantil – DCNEI] (BRASIL, 2010, p. 14), the child should be considered a

> historical and rightful subject who in the interactions, relationships and daily practices he experiences builds his personal and collective identity, plays, imagines, fancies, desires, learns, observes, experiments, narrates, questions and constructs meanings about nature and society, producing culture (our translation).



Santos (2012, p. 235) argues that "this way of seeing children can teach not only to understand them but also to see the world from the point of view of childhood. You can help us learn from them". In order to do so, we must "remove ourselves from the images that produce childhood at a time other than the present, which place it or in the future, a-to-be, an adult project; or in the past - reminiscent of a lost time of innocence of pleasure" (VASCONCELLOS, 2008, p. 74). Such ideas come from the Sociology of Childhood which, since the 90s of the last century, seeks to "rescue childhood from perspectives that understand it as an unfinished being, a maturational period of human development that is constructed independently of its historical and social conditions of existence" (VASCONCELLOS, 2008, p. 77-78).

Children are, therefore, "beings with agency", with the capacity to "act in the world, performing actions, transforming himself/herself and transforming the world itself" (SANTOS, 2012, p. 236). This conception of childhood and children has many implications for early childhood education. According to Vasconcellos (2008, p. 83),

> there is no place for educational practices in which the teacher speaks and the children listen; the teacher rules and the children obey; the teacher interprets and the children agree; the teacher gives direction and the children follow; the teacher imposes the times and spaces of the routine and the children adapt […] On the other hand, there is also no place for practices that subordinate work to children's wants (or restrict) educational experiences to their educational universe (our translation).

The conceptions of authors who defend the quality of education of Brazilian children, as well as the national guidelines of this first stage of basic education, lead us to think about the role of teachers and their training to work in early childhood education.

> The teacher is very important because he/she plays a preponderant role in mediating children's relationships with nature and culture by choosing the themes, activities, situations, materials, to enable the children's discovery, the interactions and dialogues between them, articulating the different knowledge and favoring the development of the different artistic - cultural languages (VASCONCELLOS, 2008, p. 84, our translation).

However, in Brazil, the training of early childhood school teachers has proved insufficient to provide theoretical and practical support. A teacher education program for early childhood education needs to discuss the political and pedagogical role of the child educator so that the child is guaranteed the right to childhood and to quality education. And, the right to live the childhood in a school that respects its singularity and that promotes a pedagogical work in different fields of experiences and knowledge (CRUZ, 2010; HORN, 2016; OLIVEIRA, 2009). This is one of the great challenges for public and private universities, for undergraduate courses and for teachers who train future teachers. For it presupposes offering an initial formation that is not restricted to the theoretical and technical aspects, usually disconnected from the problems and social demands of the educational reality itself.

The activities of teacher training and research can make possible a relationship between theory and practice, so that future teachers can articulate scientific and pedagogical knowledge to school life while developing their social responsibility with the academy, school and community. To be attentive to the life that happens in the school, not only to produce training report, but to problematise this reality, is a posture of a critical-reflective teacher and researcher (PIMENTA; LIMA, 2012; SANTOS-WAGNER, 1997). An educator who uses research as an epistemological and methodological tool for the process of re-elaboration of knowledge and of "other ways of practicing education in the schools of Early Childhood Education" (HORN, 2016, p. 37).



> Research during training as a method for training future teachers, translates, on the one hand, into the mobilisation of research that allows the expansion and analysis of the contexts where the training takes place; on the other hand, and in particular, it translates into the possibility of trainees developing posture and researcher skills from internship situations, elaborating projects that allow them at the same time to understand and problematise the situations they observe (PIMENTA; LIMA, 2012, p. 46).

## *Thinking Internship, Research and Geometry in Early Childhood Education*

Geometry and spatial thinking are considered crucial for children's mathematical learning. Unfortunately, according to studies by Clements and Sarama (2011), they are often ignored or minimized in early childhood education and teacher training. For these authors, geometric knowledge is highly related to mathematical reasoning and a host of other mathematical concepts and abilities, including proportional reasoning, judicious application of knowledge, concepts and properties, and data management and processing skills. They argue, therefore, that geometry can be a gateway to advanced mathematical thinking skills.

Clements and Sarama (2011) argue that children can benefit from attention to geometric and spatial thinking since early childhood education. Besides the intimate relationship with mathematics, the work with spatial abilities in childhood is articulated to learning in other domains of knowledge such as computer graphics, navigation, geography, visual arts and architecture, among others. Unfortunately, according to the authors, international studies indicate a fragility in the geometric achievements of students since

kindergarten. They emphasise that it is necessary to invest in teacher training to cope with students' limited geometry learning so they can interpret the world and solve mathematical and everyday problems.

In Brazil, studies by Smole (2003; 2014) and Lorenzato (2010) also recommend geometry in early childhood education, involving body development, spatial localisation and knowledge of geometric figures. Smole (2003, p. 105) states that "the child first encounters the world and makes explorations to progressively create forms of representation of this world: images, drawings, verbal language". It is necessary to work with the main spatial skills presented in Frame 1.

**Frame 1.** Spatial skills to be developed

| Ability | Description |
|---|---|
| **1. Visual discrimination** | Perceive similarities and/or differences between three-dimensional objects or between drawn figures. |
| **2. Visual memory** | Ability to remember what is no longer in your sight. |
| **3. Field decomposition** | Ability to isolate the visual field in its parts or assemble the whole by joining its parts. |
| **4. Conservation of shape and size** | Perception that objects have invariant properties. |
| **5. Visual-motor coordination** | Ability to look and do simultaneously. While performing one or more activities, the look and do are required at the same time. |
| **6. Motion Equivalence** | Perception of equivalence between two figures in different positions when performing translation, rotation and reflection movements. |

Source: Adapted from Lorenzato (2010) and Smole (2003).



Intentional instruction on the part of teachers and the school can contribute to the development of these skills (CLEMENTS, 2004; CLEMENTS; SARAMA, 2011). We emphasise that there is no need for a formal mathematics and/or geometry class for children in early childhood education. At school, there are plenty of opportunities to listen, see, understand, and develop spatial skills in playful, enjoyable and creative ways, whether in classrooms, cafeterias or parks. In this context, we find guidelines in the National Curriculum Guidelines for Early Childhood Education [Diretrizes Curriculares Nacionais para a Educação Infantil – DCNEI] (BRASIL, 2010) for working with children in relation to the organization of space, time and materials, ensuring "the large movements and movements of children in the internal spaces and external to the reference rooms of the classes and to the institution" (BRASIL, 2010, p. 20). Through "jokes and experiences" that "recreate, in contexts that are meaningful for children, quantitative relationships, measures, forms and spatio-temporal orientations" (BRASIL, 2010, p. 26).

Recently, the final version of the National Curricular Common Base [Base Nacional Comum Curricular – BNCC] (BRASIL, 2017) was published. In this document, we find a link between the "jokes and experiences" axes indicated in the DCNEI (BRASIL, 2010), the learning rights of children in children's education (live, play, participate, explore, express and

get to know each other). It also relates the said axes and learning rights to the fields of experiences to be developed with the children: the self, the other and us; body, gestures and movements; traits, sounds, colors and images; listening, speaking, language and thinking; and spaces, times, quantities, relations and transformations. In relation to this last field of experience, we find that

> children live in spaces and times of different dimensions, in a world made up of natural and sociocultural phenomena. Since they were very small, they seek to be in different spaces (street, neighborhood, city etc.) and times (day and night, today, yesterday and tomorrow etc.). Moreover, in these experiences and in many others, children often encounter mathematical knowledge (counting, ordering, relationships between quantities, dimensions, measures, comparison of weights and lengths, evaluation of distances, recognition of geometric forms, knowledge and recognition of cardinal and ordinal numerals, etc.) that equally sharpen curiosity (BRASIL, 2017, p. 38, our translation).

We emphasise in these official documents the need to work with space skills in child education, in an intentional and planned way by the teachers. Here is a great challenge for every teacher in every Brazilian school of early childhood education. According to Clements and Sarama (2011), most teachers had few experiences with geometry in their school and professional lives. Therefore, it is not surprising that geometric instructions are limited in classrooms. They emphasise, therefore, the need to promote a curious and in-depth study of geometry in the training of teachers who teach mathematics in early childhood and early childhood education.



In this article, we discuss one of the spatial abilities indicated by Clements (2004), Lorenzato (2010) and Smole (2003) and worked with children of a kindergarten class during the probationary period: composition and decomposition of figures, or either the ability to isolate the visual field in its parts or to assemble the whole by joining its parts. Based on research carried out with children between 4 and 8 years old, Clements (2004) states that they usually present a non-chronological and linear learning trajectory at different levels (Frame 2). It stresses that being aware of these levels can serve as a guideline for educators and researchers, but not as a fixed formula and equally applicable in differentiated contexts.

We highlight, as the author, that we do not think of these levels in a predictable, linear and obligatory way. Once triggered, the processes can change course, with different intensities in each child. We consider it relevant that adults take advantage of the moments of interaction with the children to dialogue, to try to understand what they express and think about the lived experience. These conversations need to occur at different times so that they have opportunities to (re) build and internalise ideas and arguments they exchange with other children and adults (VYGOTSKY, 2001). We underline the importance of these processes in the construction of ways to learn mathematical/geometric knowledge that is essential to children's lives.

**Frame 2.** Levels of composition and decomposition of figures

| Levels | Description |
|---|---|
| **1. Precomposer** | Manipulation of figures as "unique", with no interest in combining them for composition in a larger way. Use, for example, one piece to represent the sun, another for the tree, etc. |
| **2. Piece Assembler** | Combination of figures by trial and error for composition of other images. But, as in the precomposer level, each figure or part has a unique function. Visualization of few geometric relations between the parts. Still limited use of rotational, translational and reflection movements. |
| **3. Picture Maker** | Concatenation of several figures by trial and error in order to construct other images. For example, a leg can be created from three contiguous squares. There is still no anticipation of the formation of new geometric figures. There is already the correspondence of corners, but not the knowledge of angle as a quantitative entity. Rotation and inversion are also used by trial and error to select and discard figures. |
| **4. Shape Composer** | Intentionality and anticipation in the composition of new figures based on the attributes of each one. Use of angle analysis as well as lateral lengths. Rotation and inversion movements are also used intentionally. |
| **5. Substitution Composer** | Deliberate composition of new figures or new units with recognition of their attributes. Substitutions are made, for example, intentionally joining two triangles to form a diamond and vice versa. |
| **6. Form Composite Iterater** | Construction and operation with new figures or units formed by composition. Using the iteration in a deliberate, systematic and intentional way. |
| **7. Shape Composer with Superordinate Units** | Besides the construction and operation with new composite figures, there is already an intentional process of ordering and hierarchising these new units. |

Source: Adapted from Clements (2004).



## Composition and decomposition of geometric figures: data analysis and discussion

Based on the participant observation and conversations with the teacher of the children's education class and with the children, one of the groups of undergraduates thought about carrying out a research related to geometry in early childhood education to be held in the semi-boarding school in Sergipe.

> When I asked about the math content that was worked on, the teacher talked about numbers, counting, and geometric shapes. The teacher said that the children already knew how to count, add, and also squares, rectangles, triangles and circles […]. I asked what activities the children did with these forms. She said that "they were more than painting". For example, "paint the squares of one color, the circles of another, the triangles of another". The shapes were fixed on the walls of the room, when the children arrived, they played jigsaw puzzles, or with logical blocks. We played with them and made up stories with those toys. I thought we could enjoy those moments more. But during the weeks I've been there, no task was done with a focus on geometry. We could do something (Maria Field Diary, student of the Pedagogy of UFS, fictitious name).

And we did. We thought together a geometry teacher training/research for children. The data led us, as teachers and researchers, to problematise this experience. We therefore

recall the question that drives us in this article: What contributions does this articulated research bring to the training of early childhood school teachers and, in particular, to the field of geometry in childhood? Thus, we bring here analyses and reflections of one of the proposed tasks for composition and decomposition of geometric figures.

The selected activity was to present children, circles, squares, rectangles and triangles divided into two, three or four parts to be assembled and disassembled, in order to favour the composition and decomposition of figures. This task was applied with the 18 children of the class and here we present the analysis of a group of four of them: Sophia, Mateus, Ricardo and Samuel (fictitious names). It should be noted that although the activity focuses on the ability of figure composition, other spatial abilities such as visual discrimination, visual-motor coordination, visual memory, movement equivalence (Frame 1) also manifested during the task.

"Could we put this puzzle together?" That's how we introduced the activity. The children were eager to receive the parts and begin the assembly. They sat in groups, showing interest in assembling/disassembling the figures. We observed that all the children interacted at the time of this activity. In general, they have been able to (make up) the whole from their parts.



> Matthew and Samuel, both five years old, sat with their colored parts and soon began to join them. They talked about the montage: "That part is not from there, no ... can't you can see that it has two smaller sides, and the top and bottom is bigger?" Sophia, 5, easily mounted the triangle, divided into 02 parts and the circle into 04 parts. She had some difficulty with the square in 3 parts, but after trying a few times, turning, dragging, and changing the pieces of place, she concluded: "Ah ... it was a square!" Ricardo, aged 4, at first started to assemble the figures, but soon he was building castles, houses, cars, etc. Sophia, who had already finished, when she saw Ricardo playing, shouted: "Ricardoooo, come on, I'm going to take mine!" And Sophia left to create castles and stories (Field Diary, traineeship teacher).

The children performed sliding or rotating movements in the attempt to assemble the objects, also operating with the ability of movement equivalence (LORENZATO, 2010; SMOLE, 2003). According to Lorenzato (2010), children can move one figure over the other or side by side, and this movement can be of three types: a) translation: when all points of the figure move in the same direction , when we open a drawer, a sliding door, etc.); b) rotation: when the figure rotates upon an axis (for example, a top, a fan, hands of a clock); c) reflection: when a mirror image occurs (for example, observe the image of your right hand and see that it appears to be your left hand).

For the composition of the figures, the children of the semi-boarding school used trial and error processes (CLEMENTS, 2004), signaling to the picture maker level, because in their attempts they turned and inverted the parts, as well as organised the figures by their corners. They also made comparisons and correspondences between the spaces and the figures that filled them. They worked, therefore, with visual discernment, perceiving similarities or not between the boundaries of the figures (LORENZATO, 2010; SMOLE, 2003).

The children's movements and speeches indicated these spatial abilities. Sophia, when finding that "it was a square", visually discriminated the object when comparing, classifying and naming the form. The dialogues between João Victor and Samuel evidenced their problematisation, verifying the non-correspondence of the parts of the objects that they tried to mount. The expressions "two sides smaller and the lower and uppermost part" - pointed to the internalisation of features of the rectangle and the recognition of the shape by its attributes (CLEMENTS, 2004; VYGOTSKY, 2001). The presence of important notions as smaller/bigger, from on/under can be considered the starting point for an intentional pedagogical work with spatial abilities and other subjects (LORENZATO, 2010; MENDES; DELGADO, 2008).

However, it becomes important to look at visual prototypes that children construct, in this case, on geometric figures, as well as at the teachers' need to question these prototypical images. In general, when we ask a person to draw or form a quadrilateral or a triangle, what appears is a square or an equilateral triangle. These are examples of what is considered an established visual prototype (HERSHKOWITZ, 1994). Clements (2004) and Clements and Sarama (2011) presented several studies with hundreds of children between 3 and 6 years old, in which the circles, squares, rectangles and triangles were recognised and named with some ease and related to a visual prototype.



Regarding the squares, the authors reported that the children in the research were more likely to identify them when their justifications were based on attributes such as, for example, number and length of sides. As for the rectangles, the relationship between height and base has become fundamental for the identification of these figures by children. In general, in research tasks, they rejected figures they considered "too thin" or "not long enough". For Clements (2004), these results indicated the potential of children to learn geometry. But they also pointed to a fragility of educational practices that worked with what they had already built.

In the case of our study, we analysed that the children's speeches and the way they moved the pieces to compose the geometric figures indicated both their potentialities and a fragility of educational practices. We want to say that the children of our research showed signs of initial geometric knowledge, moved the figures (rotation, translation and reflection) in order to find a way of composing or decomposing. However, the class teacher and the students in the Pedagogy course did not question how students would handle the pieces according to the established visual prototype. The movements made by the children always had as reference the image of the figures (squares, rectangles and triangles, except for the circles) in their prototypical position in the horizontal base.

We stress here the essential role of the teacher in observing and listening to the children in these school moments. Such situations may constitute opportunities for dialogue with children and approximation to what they are thinking. Thus, from what they observe, listen and dialogue with the children, teachers can plan actions that aid in the internalization and learning of mathematical concepts, as already indicated by Vygotsky's (2001) studies.

> With those small hands, Ricardo sets out to create other forms. He changes the place of the pieces, flips them over and tries again. He joins the corners, even

> without the mathematical knowledge about angles. Until he is satisfied, at least momentarily. And he shouts, "Tiaaaaaa, I made a lane!" (Field Diary, teacher of the internship discipline).

We relate this episode to the levels of composition and decomposition presented by Clements (2004). It seemed to us that Sophia and Ricardo were predominantly in the third level (picture maker), when trying to compose figures from the parts. We note the following characteristics, from the text of Clements (2004, p. 277):

> New configurations are chosen […] The child can try to match corners, but angle for her is not a quantitative entity […]. Rotation and inversion are used, usually by trial and error, to try different arrangements (the "pick and drop" strategy). Thus, they can complete a framework that suggests the placement of individual forms, but which, together, can play a single semantic role in the image.

Another important aspect to highlight was the interaction between the children of the semi-boarding school. According to Vygotsky (2001), when interacting with their peers and adults, children internalise knowledge, roles and social functions that influence the formation of knowledge and the development of one's own consciousness. This interaction was present in children's speech as they exchanged ideas with other peers and between themselves, and expressed what they were thinking and doing at the same time. Expressions such as "that part is not from there", "you cannot see that there are two smaller sides [smaller], and the top and bottom are larger" contribute to these children thinking about their own thinking, which is fundamental to their mathematical learning.



In short, an approach to early childhood education in an articulated perspective of teacher training and research, and with an emphasis on mathematics/geometry for children, has brought important reflections both for teachers in initial teacher education and for us as teacher educators and researchers. The reflection of one of the students makes us think about this:

> It has not been easy to work with training and research together and still with geometry, don't even mention it. I discovered that I know so little and was not prepared ... I learned a lot and started studying space skills, I had no idea that this had to do with geometric shapes. And there's something else, the kids, what they say, the people need to pay more attention. It is a wealth of knowledge! (Ana's field diary, student of the Pedagogy of UFS, fictitious name).

## Final considerations

When we reflect on the produced data, we can infer that: a) the children demonstrated knowledge about geometric figures related to established visual prototypes; b) their actions in manipulating parts of the figures seemed to indicate the picture maker level pointed out by Clements (2004); c) similar activities can be used and accompanied by teachers and/or researchers in order to analyse the children's geometric thinking and to think an effective pedagogical work from such reflection; and d) these cited aspects can mark future and necessary research on geometry of children, an issue not discussed in Brazilian academic production.

We show that, as researchers and teachers' education, we broadened our understanding from in-depth readings of the subject, which clarified the data and indicated the potentialities of research of this nature. We reiterate the importance of articulating teacher training and research as an opportunity to "think scientifically" from school. This articulation makes it possible for future teachers to question, raise hypotheses, select procedures, produce data and reflect on the research process. We also emphasise the importance of observing and listening carefully to children in their interactions with other children and adults. Often, in the effort to comply with curricular proposals for early childhood education, teachers end up losing the wealth of knowledge between conversations, silences, ways of solving situations, movements and gestures. We do not advocate one posture over another. We underline the importance of going through **a many-hand path**, including those of children, in which we need to learn to walk.

We learned that going to school gave those future teachers an experience of articulating theory and practice, practice and theory, besides an encounter with gaps regarding the field of geometry. It made possible to think about investigative questions addressed to these gaps found in the school and in the knowledge and teacher training itself. Finally, we underline how beneficial was the fact of analysing the data produced by the research in the light of new references. The past and lived time became a wise friend for a theoretical deepening and corroborated for current and ongoing reflections on teacher training, research and geometry in early childhood education. As Horn (2016, p. 49) expresses, "we cannot repeat old practices of a doing that already exists, but open intensities for the new, for creation [...] from the questions brought by the students, we need to be always attentive to restructure teaching practices in the form of a supervised training".



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