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ARTICLE

Does the order of the factors change the result?[i]

A ordem dos fatores não altera o resultado?

¿El orden de los factores cambia el resultado?

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Abstract: The article presents an investigation that used mathematical games proposed by the National Pact of Literacy at the Right Age (PNAIC). The objective was to analyze episodes related to the multiplicative field, referring to the game "A boot of many leagues". The researcher's reflections are presented, alluding to the experiences provided by the use of the game. Field diaries, photographs, audio and video recordings were used. As a result, the analyzes on the approach of the multiplicative field indicate the potential of the use of pedagogical practices supported by games to expand the understanding of the referred content by the children, through the mediation of the teacher, which contributed to the resolution of the problem situation presented. Still, it was verified the importance of identifying the multiplier and the multiplicand, although the commutativity, present in the multiplication, does not change the result of the operation.

Keywords: Mathematical literacy. Numbers and operations. PNAIC.

Resumo: *O artigo apresenta uma investigação que utilizou jogos matemáticos propostos pelo Pacto Nacional pela Alfabetização na Idade Certa (PNAIC). O objetivo foi analisar episódios relacionados ao campo multiplicativo, referentes ao jogo "A bota de muitas léguas". Apresentam-se reflexões da pesquisadora, alusivas às experiências proporcionadas pelo uso do jogo. Utilizaram-se diário de campo, fotografias, áudio e videogravações. Como resultado, as análises sobre a abordagem do campo multiplicativo indicam a potencialidade do uso de práticas pedagógicas amparadas em jogos para ampliar a compreensão do referido conteúdo pelas crianças, a partir da mediação da professora, o que contribuiu para a resolução da situação-problema apresentada. Ainda, verificou-se a importância da identificação do multiplicador e do multiplicando, apesar de a comutatividade, presente na multiplicação, não alterar o resultado da operação.*

Palavras-chave: *Alfabetização matemática. Números e operações. PNAIC.*

Resumen: *El artículo presenta una investigación que utilizó juegos matemáticos propuestos por el Pacto Nacional por la Alfabetización en la Edad Adecuada (PNAIC). El objetivo era analizar episodios relacionados con el campo multiplicativo, refiriéndose al juego "Un arranque de muchas ligas". Se presentan las reflexiones del investigador, aludiendo a las experiencias proporcionadas por el uso del juego. Se utilizaron diarios de campo, fotografías, grabaciones de audio y video. Como resultado, los análisis sobre el enfoque del campo multiplicativo indican el potencial del uso de prácticas pedagógicas apoyadas por juegos para expandir la comprensión del contenido referido por parte de los niños, a través de la mediación del maestro, que contribuyó a la resolución de la situación problemática presentada. Aún así, se verificó la importancia de identificar el multiplicador y el multiplicando, aunque la conmutatividad, presente en la multiplicación, no cambia el resultado de la operación.*

Palabras clave: *Alfabetización matemática. Números y operaciones. PNAIC.*

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Introduction

This article refers to an excerpt of the master's research, in which the professor-researcher investigated her own practice in the classroom in which she worked during the research, in a municipal school in the city of Campinas, SP, in 2018. The objective, the general purpose of this study was to investigate what knowledge was developed by the teacher-researcher, based on the use of games to learn numbers and operations with children from the 3rd year of elementary school, seeking to analyze the educational actions that contributed to this process and transformations in teaching practice.

The games were chosen because the teacher-researcher sought to study resources that could arouse the students' interest and motivation, considering that "the use of games implies a significant change in the teaching-learning processes, which allows to change the traditional model teaching, which often has the main resource in the book and in standardized exercises" (SMOLE; DINIZ; CÂNDIDO, 2007, p. 11).

The professor-researcher is a participant in the Study Group Teachers Mathematizing in the Early Years - GEProMAI -, which meets weekly at the Pontifical Catholic University of Campinas (PUC-Campinas). The studies in this group motivated the investigation of the theme, as well as the participation of the teacher-researcher in the formative process of the National Pact of Literacy at the Right Age (PNAIC), in 2014 (BRASIL, 2014a).

Such participation brought significant contributions to the training and performance of the teacher-researcher, who at the beginning of her career as a teacher felt insecure when teaching mathematics classes for her classes in the early years of elementary school. I had no mastery of the content to be taught and, even less, of strategies to teach it. It was based on memories of how he had learned at school, on the textbooks in the teaching units where he worked and on the exchange of experiences with his colleagues.

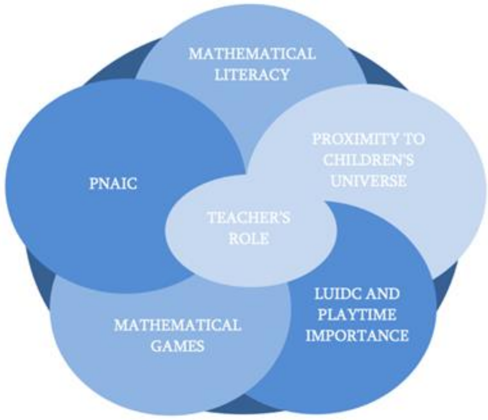
Indeed, Megid (2015) highlights the challenges experienced by teachers who teach mathematics in the early years, the same ones that the teacher-researcher faced: insecurity, the reproduction of actions experienced in childhood and, also, the lack of time for planning and studies.

PNAIC was important for the training of the teacher-researcher, allowing her to explore new resources and understand, in other ways, certain mathematical concepts. With the new training, he began to be more concerned with the teaching and learning process of students and with the way of teaching. Finally, the program provided moments of reflection and exchange of experiences among teachers.

Her teacher-researcher training process since the beginning of her teaching career, her participation in PNAIC and GEProMAI, her search for strategies to arouse the interest of students and the improvement of teaching work, were the motivations that led her to research her own practice.

The image in Figure 1 was produced by the teacher-researcher to indicate the main aspects related to the development of the research.

Figure 1. Aspects related to research development



Source: Elaboration of the authors.

Mathematical literacy at PNAIC is anchored in the perspective of mathematical literacy, considering the social aspects of mathematics, which go beyond the domain of codes and operative techniques. In this perspective, Passos and Nacarato (2018, p. 123) consider mathematical literacy as "a mathematical education that values students' knowledge and helps them understand the ways in which our society organizes their experiences with support from Mathematics, promoting understanding and reading the world".

Thus, for the teaching of mathematics, the program indicates the approach to the child's universe, emphasizing the importance of playfulness and playing. In this way, it presents as one of the work possibilities the use of mathematical games, which, according to Nacarato, Mengali and Passos (2009) encourage the active participation of students in the construction of knowledge and in the construction of meanings of mathematical concepts. The PNAIC also indicates the importance of the teacher's role for the mathematical literacy process, as emphasized by Nacarato, Mengali and Passos (2009), who attribute to the teacher the main role for the constitution of an environment that promotes learning, mainly in what refers to the choice of activities and the method to be used. In this case, the path chosen by the teacher-researcher to develop the investigation was the use of games, which seems adequate to work with students of the early years, a fundamental step for the understanding of several basic concepts necessary to reach more complex ones, even though the teaching of mathematics is often still guided by memorization and model copying, encouraging the student to just find the right answers, without understanding the concepts involved.

In order to promote meaning in the teaching and learning process of mathematics, two games proposed by the PNAIC, of the numbers and operations axis, were chosen for this investigation. The selected games, according to Muniz (2014), must be proposed by the teacher and used as a didactic activity, following the rules previously taught. "The teacher, in this case, is the creator, prescriptor and controller of the ludic activity" (MUNIZ, 2014, p. 129). In this perspective, the research is also anchored in Grando (1995, p. 52), who conceptualizes as pedagogical games those that "have their pedagogical value, that is, that can be used during the teaching and learning process".

Therefore, the games used in this research have pedagogical intent and aim at learning mathematical concepts, aiming at learning important concepts in the field of numbers and operations, including the positional value and the multiplicative field.

This article aims to present the analysis of some episodes of this research, which refer to the game "The boot of many leagues". The proposal turned to work with problem situations in the multiplicative field and provided to reveal the reflections of the teacher-researcher from the use of this game. So, let's move on to the presentation.

Initially, considering the "numbers and operations" axis, it is important to characterize the sense of number as "something imprecise, personal and personalized, which is related to the ideas that each one has been establishing about numbers and operations and which is not always easy to describe" (CEBOLA, 2002, p. 226). In the same direction, Van de Walle (2009, p. 148) highlights that the evolution of numerical sense is something constant, "it develops when students understand the size of numbers, develop multiple ways of thinking about and represent numbers, use numbers like and develop precise perceptions about the effects of operations on numbers". It is expanded, when children begin to "use numbers in operations, build an understanding of positional value and develop flexible methods for calculating and making estimates involving large numbers" (VAN DE WALLE, 2009, p. 148).

The development of the numerical sense makes it possible to understand the Decimal Numbering System, with its main characteristics: "the decimal base, the positional notation and ten signs that make it possible to represent any number", from 1 to 9 and zero, which refers to "empty or white space"(TOLEDO; TOLEDO, 2009, p. 62).

It is also necessary to highlight the differentiation between additive reasoning and multiplicative reasoning, as indicated by Nunes *et al.* (2009, p. 84): the additive reasoning is the part-whole relationship. When you want to "know the value of the whole, we add the parts together; if we want to know about one part, we subtract the other part from the whole; if we want to compare two quantities, we analyze what part of the larger quantity is left if we remove an amount equivalent to the other part". Such practices involve several actions, such as joining, separating and making correspondences, pointed out by the authors as action schemes associated with the additive field.

It is related to the multiplicative reasoning, thus defined by Nunes *et al.* (2009, p. 85): "it is the existence of a fixed relationship between two variables (or two quantities or quantities). Any multiplicative situation involves two quantities in constant relation to each other". The authors argue that children, from an early age, perform multiplication in a practical way, using the one-to-many correspondence action scheme, using additive reasoning and the equitable distribution scheme in problems involving division.

Having presented the principles, it is appropriate to indicate the method used in the investigation and its characteristics.

Trailed road

The investigation was configured in a research of an interventional nature and of the practice itself, since the researcher assumed a double role: teacher and researcher. Teixeira and Megid Neto (2017, p. 107) point out the research of their own practice as belonging to a list of interventional studies and affirm that "research in this area is involved in processes of understanding and improving teaching work, starting with reflection about their own practice and professional experiences".

Lima and Nacarato (2009) indicate two movements that configure the research of their own practice: the first refers to the teacher who voluntarily participates in collaborative groups, discussing problems that arise in his classroom; the other refers to the professor who enters a postgraduate program and investigates his own practice. Both movements were experienced by the teacher-researcher and contributed to the process of reflecting on her actions.

Ponte (2002, p. 3) presents three justifications for conducting the research of the practice itself, in which this study is anchored in. The first refers to the teacher, who in this type of research becomes the protagonist of curriculum and professional development. The second concerns the research capacity to enhance the professional development of the teacher and to act in the school culture, with possibilities to transform it. The third justification is related to the fact that research on practice contributes to elements that provide a greater understanding of educational problems and professional culture. According to the author, this type of research involves real situations, of teaching practice, with problems normally experienced by the teacher and researcher. It has the objective of intervening and transforming, preliminarily understanding the research problems and, in a second moment, activating more appropriate action strategies.

Once the nature of this study was characterized, we started to consider the research instruments: the use of a game proposed in the training material of the PNAIC - "The boot of many leagues" - will be explained here, with the description of the observations, the oral reports and written, the problematizations proposed by the teacher-researcher before, during and after the use of the games. For the record, a field diary, photographs, audio and video recordings were used. The research was carried out by the following route:

Figure 2. Search path

Source: Elaborated by the authors.

The initial and final research stages consisted of a dialogue between the teacher-researcher and the class and the students' writing about what they thought about math classes, before and after using games, configuring moments that helped in the process of reflection on the teacher's practice.

The game was played at three different times, as indicated by the PNAIC, in the “Game Notebook on Mathematical Literacy” (BRASIL, 2014b). The first was concerned with the presentation of the game, the materials used and the rules and with the division of the teams. The second dealt with the application of the games, properly speaking. The third included moments of socialization, impressions, reflections on what was learned in mathematics and oral and written problems.

The experiences with the game “The boot of many leagues” reported here had the learning objective of “elaborating, interpreting and solving problem situations in the multiplicative field (multiplication and division), using and communicating their personal strategies through different languages and exploring the different meanings” (BRASIL, 2014c, p. 33). This document also states that this game aims to “develop the idea of multiplication and division when calculating the number of jumps that the “boot” will make; use zero as a starting point reference” (BRASIL, 2014c, p. 33).

In practice with the game, two variables necessary for the multiplication are presented: the number of hops and the size of them. In order to develop activities with this game, the PNAIC “Game Notebook on Mathematical Literacy” (BRASIL, 2014c) indicates that the teacher must draw the graduated line on the floor - use a graduated paper strip - with the marks from 0 to 25. The notebook proposes the use of blue and yellow tokens, from 0 to 5, one color for the size of the jump and another color for the number of jumps. The student draws a card of each color and calculates which number will arrive on the line. In her practice, the teacher-researcher made an adaptation: instead of the colored cards, she used two dice, one for each objective, that is, one for the size of the jump and another for the number of jumps to be given by the player.

From the presentation of these episodes, this article aims to present the reflections related to the multiplicative field, carried out by the teacher-researcher on the teaching of mathematics in the early years, involving the ideas of multiplication and division. However, although it is a research of the practice itself, which relied on the reflective processes of the teacher-researcher throughout the development of the study, actions were also recorded in partnership with the advisor, also the author of this article.

The research was approved by the Ethics Committee of the Pontifical Catholic University of Campinas, under Opinion number 2,907,681. It also had the authorization of the students and their guardians, for signing the terms of free and informed consent and consent.

The route of the game

This report will allow the reader to follow the children's plays throughout the activity and, to preserve their anonymity, they were identified by the letter C, accompanied by a number, that is, the inscription C1 will be used to always indicate the same child, the even occurring with all the others. The letter Q is used to indicate the insertions of the questions from the teacher-researcher.

The teacher presented the game to the students and took them to the court, where she had inscribed a line with markings from 1 to 25 - each one would correspond to a jump by the children. Some moves were selected to be presented in this article, those useful for the analysis of the work with the multiplicative principle.

Initially, we highlight student C26's move. The quantity data fell in number 3 and, in relation to size, number 4:

> C12: *First, she will have to fall on three.*
> Q: *The jump size is four.*
> C12: *So, from zero to 3.*

The teacher went to the straight, demonstrated the distance of size 4 and continued walking and adding:

> Q: *From zero to 1, we have one; from 1 to 2, we have one more, which gives 2; from 2 to 3, we have one more, which gives 3; and from 3 to 4, we have one more, which gives 4. So, you will arrive at 4, not 3.*
> C12: *I thought it dropped to 3 because zero counts.*

The student went to the straight and counted the zero as if it were the one.

> C12: *1* (referring to zero), *2* (referring to 1), *3* (referring to 2) *and 4* (referring to 3).

The teacher returned to the line and repeated the explanation, adding 1 in 1, starting from zero (1 + 1 + 1 + 1 = 4). He repeated that zero was the starting point and showed that from zero to 3 the total was 3, not 4.

C12: *She will do like the C16, but* [with] *two less jumps. It will decrease two.*
Q: [To] *C16, there were five jumps out of 4; and* [for] *C26, there are three jumps of 4. What number will it arrive at?*
C22: *12.*
Q: *C22 said 12, is that right?*
C7: *From zero to 12.*
Q: *And why 12?*
C22: *0 + 4 = 4; 4 + 4 = 8; 8 + 4 = 12.*
Q: *Yes, three jumps out of 4; 4 + 4 + 4 = 12. Do you have another account that you could do?*
C22: *Of times?*
Q: *This can be 3 x 4, which is the same as 4 + 4 + 4, and the result is the same.*

The student started the jumps, and her colleagues helped in the counting, adding 4 in 4 until reaching 12. When the student reached 12, the teacher-researcher asked her to come back and take the route, now returning 4 in 4. The other students helped to count (12 - 4 = 8; 8 - 4 = 4; 4 - 4 = 0).

In this dialogue, the discussion of zero as a starting point emerged. Student C12 represented quantity 4, starting from zero, but assigning him "1". The representation of the line on the floor helped in the intervention and allowed the teacher-researcher to demonstrate to the student that the starting point is zero, differently from what he had done. He emphasized that, from zero to 1, we have the distance "1", and so on, successively. The reflection on the difficulty of some children to understand the meaning of "zero" became more evident, for the teacher-researcher, in this activity. Reflecting on children's actions is not always common practice in everyday life, especially in activities related to mathematics. Sometimes, the indication of success or error is attributed, without seeking the paths, the intentions indicated by the children.

The students were unable to directly establish the multiplication relationship between the two terms. Only C22, a student who performs well in mathematics, had this perception. At first, he performed the sum of equal plots, still referring to the additive field, using the part-whole relationship, which is established by adding the parts to know the whole, as found in Nunes *et al.* (2009). Then he explained his reasoning himself, using the multiplicative field. According to Nunes *et al.* (2009), this is the relationship between two variables - in this case, between the number of jumps and the distance covered.

The intervention of the teacher-researcher was necessary for the student to explain his reasoning and for the others to begin to understand that it was possible to perform a multiplication to calculate the distance traveled, which contributed to the expansion of the numerical field and the numerical sense.

Then, C23 played, and five jumps of length 5 were drawn.

Q: *Five jumps out of 5: what number of the line will you reach?*
Student C23 did not respond.
C3: *Her result is 25.*
Q: *C3 said it is 25; isn't it? What do you think? The team can help C23.*
Student C16 asked to count on the straight and was simulating the jumps that the C23 would have to do.

Q: *Shall we help you?* [With] *the first jump, will she start from scratch and go to where?*
Several: *5!*
Q: *And what next?*
Several: *10!*
Q: *And the third jump?*
Several: *15!*
Q: *And then, the fourth jump?*
Several: *20!*
Q: *And the last jump goes to ...?*
Several: *25!*
Q: *Is that so, guys?*
C3: *5 x 5 = 25.*
Q: *Yes: Five groups of 5 is the same as 5 x 5. Come on, C23!*

In this play it is noticeable that C3 understood the concept of multiplication involved in the game. He established the relationship between the two variables of the two data and performed the multiplication, unlike C23, which had difficulties in mathematics and still could not understand the multiplicative field. With C23 it was necessary to intervene, demonstrating the groups of 5 and indicating that five groups of 5, in the additive field, is like performing five times the 5.

Cebola (2002), anchored in McIntosh *et al.* (1992), presents ideas that lead to the characterization of the meaning of the basic number, among them, knowledge and dexterity with numbers, including the understanding of their various representations, which allows us to understand that 5 + 5 + 5 + 5 + 5 it is the same as 5 x 5. C23, as was exemplified by lines before, had not yet understood the relationship of these representations, unlike C3. Intervention and diary soon provided by the game, both with the teacher-researcher and with colleagues, allowed the active participation of C23. She accompanied the student and asked her to add 5 to each jump. The student used her fingers as a support for counting.

In fact, fingers are relevant instruments to assist in counting. Muniz, Santana, Magina and Freitas, in the notebook on "Construction of the Decimal Numbering System" of the PNAIC (BRASIL, 2014b), indicate the support of finger counting as an important reinforcement in the construction of the number by the child, a symbolic basis that is fundamental to this process, allowing develop counting and calculation resolution strategies. Megid (2010) also highlights the use of fingers as a "private abacus", which allows children to develop their own strategies.

In this move, the use of hands was adopted by C23, a student who still needs help to understand the basic operations of mathematics. Thus, the use of fingers was still very significant for her. C3, which demonstrates greater understanding, did not use his fingers as support anymore.

Then, student C4, after throwing the dice, had to make two jumps of size 3.

Q: *C4, in which number will you finish your jumps?*
C4: *No six: 3 + 3.*
Q: *And what other account could you do?*
C28: *Sometimes.*

> Q: *Sometimes? Like?*
> C21: *3 times 2.*
> Q: *Are there three jumps of 2 or two jumps of 3?*
> C4: *There are two jumps of 3.*
> Q: *This is two jumps of 3: 3 + 3. And sometimes, how does it look?*
> C4: *2 times 3.*
> Q: *That's right, there are two jumps of 3, and therefore, twice the 3! If there were three jumps of 2, we would do three times 2.*

The speech of student C21 - “3 times 2” -, referring to the result of the calculation related to two jumps of size 3, revealed that the student seems to understand the commutativity in the field of multiplication, as indicated by Nunes *et al.* (2009). For three groups of 2, we have the same result as two groups of 3. However, the game contributes to the understanding that it is necessary to understand the problem situation to define the multiplier and multiplying. These terms were not mentioned by the teacher-researcher, who intervened, however, in an attempt to explain that, in that case, despite the result being the same, the situation was different. In this move, there are two jumps of 3, that is, 2 x 3 or 3 + 3, which is different from three jumps of 2, which would be 2 + 2 + 2 or 3 x 2.

The teacher was not only considering the result, but trying to provoke an understanding of the process that that situation provided. With the number line, it is possible to demonstrate this reasoning in a practical way. Not only did the result matter, which proves the commutative property - the order of the factors does not alter the product -, but it was also important to understand the multiplication.

The next was C30. The dice marked two jumps of size 3, as in the previous move. The student easily made his move. The teacher-researcher took advantage of the moment to problematize the multiplicative field, but referring to the division process. When the student stepped on number 6, he asked:

> Q: *And if I divide 6 by 2, how much will it be?*
> C22: *3.*
> Student C3 observed the line and said it was 3.
> Q: *We did 3 twice, which was 6. Now, if I divide 6 by 2, you said it will be 3. C30, come back, then, making two jumps of 3.*

The teacher-researcher asked the others to observe: the three was the half of 6. She explained that they were doing the reverse operation: twice the 3 resulted in 6, while 6 divided by 2 resulted in 3. She went to the line to demonstrate.

> Q: *You said that if I divide 6 by 2, each part will be equal to 3.*

She stood at number 3 and asked them to watch. That they verify that from 0 to 3, and from 3 to 6, the size was the same, that is, 3. So, she said that they found half. With that, it was possible for the teacher-researcher to realize that division is a more complex process to be approached and that one of the difficulties of mathematics teachers in the early years, including her, is to explain in a way that students understand that division is , among other alternatives, the inverse operation of multiplication.

In this game, it was possible to find a way to demonstrate this process. The term "half" was known to students. By jumping the line in reverse, the teacher-researcher facilitated the explanation and confirmed the response of some students who had understood this reasoning; and it was also possible to resume the same action with the other students.

With support from Grando (2000), it is possible to infer that the challenges brought by the games enabled students to develop problem solving strategies. In addition, the practice helped the teacher, who, in the course of the games, found new ways to problematize concepts that are difficult for students to understand and, initially, equally difficult to explain to her.

In another move, commutative property emerged again and so it is necessary to intervene so that students realize that, although the result is the same, the calculation should be thought out according to the situation presented. In this case, the student would have to perform two jumps of size 3.

> C28: *We can do it 3 times 2 or twice 3, which gives 6.*
> Q: *The result is the same, but which of these two accounts will I use in this case? There were two jumps of 3.*
> C18: *It has to be twice 3.*
> Q: *Yes, can I do 2 twice?*
> Some: *No.*
> Q: *Not in this case, because there are two jumps of 3, not three jumps of 2.*

The importance of mediation in the action of the game, as stated by Grando (2004), is clearly perceived in this dialogue. Then C27 played and made four size 2 jumps.

> Q: [With] *the first jump, where are you going to fall?*
> C27: *On 2.*
> The student walked to number 2.
> Q: *It's a size 2 jump. You have to jump straight to 2.*
> C12 went to the straight and demonstrated, walking to the 2.
> The student added two at a time and reached 8.
> Q: *How many jumps of 2 did you do?*
> C27: *4.*
> Q: *What is the result?*
> C27: *8.*
> Q: *What account are you going to register?*
> C12: *4 x 2 = 8.*
> Q: *Do you understand, C27? You made four jumps of 2 (2 + 2 + 2 + 2, which was 8). This is the same as doing 4 x 2 = 8.*

At that moment, once again, the teacher-researcher resumed the sum of equal plots to explain the multiplication. The line made it easier to understand the size of the jump. The student used the sum to perform the count. Then C25 made two size 5 jumps.

> Q: *How is the C25 going to do?*
> C31: *It's twice 5, like that?*
> C25: *It is a jump of 5 and another jump of 5, which will give 10.*
> Q: *Yes, as the C31 said, 2 x 5. Do not forget to register.*

This move was quick, easily understood by student C25, and demonstrated a positive aspect of the use of games, which also appeared at other times: the dialogue between students and teachers contributes to the understanding of the concepts involved. Grando (2004) highlights the importance of the dialogues that take place during the games, especially the different forms of reasoning. The same occurred in the next dialogue, built by student C8, who performed four jumps of size 1.

> Q: *How many jumps are you going to take?*
> C8: *Four.*
> Q: *And how big is the jump?*
> C8: *1.*
> The student jumped straight to 4.
> Q: *What was the colleague doing right?*
> Some: *No.*
> Q: *It's okay that you need to reach 4 at the end, but there are four jumps of size 1. Let's go back.*
> C2: *First, you jump at 1 and go 1 at 1.*
> The student came back and did it correctly.
> Q: *Do you understand, C8? How many jumps did you take?*
> C8: *Four.*
> Q: *How big is it?*
> C8: *1.*
> Q: *You jumped four out of 1. How's the calculation?*
> C25: *4 x 1.*
> Q: *That's right.*

Then C2 made five size 4 jumps.

> Q: *How many will you reach?*
> C2: *If the jump is 4, first, I will reach 4.*
> Q: *The first jump to 4! Then go.*
> C2: *The next one, at 8.*
> C31: *No. 10.*
> Q: *She was doing a 4 jump, so 4 more, 8.*
> C2: *The next one is at 12.*
> Q: *And then?*
> C2: *16.*
> Q: *How many jumps are left?*
> C2: *One; will fall on the 20th.*
> Q: *What's up, guys? That's right? Did she do the five jumps of 4?*
> Some: *yes!*
> Q: *Yes, she did! How will you record the calculation?*
> C25: *More or more times.*
> Q: *If it is more, how is it?*
> C25: *10?*
> Q: *Let's think!*
> Q: *How did you think to jump? How did you tell?*
> C2: *Every four.*
> Q: *She was counting every four because the jump was four. How many times have you counted 4?*
> C2: *Five.*

Q: *Five times, she counted the four. So, what calculations can we register?*
C24: *4 x 5.*
Q: *Did she make four out of five? Or was it five jumps out of 4?*
C25: *5 x 4.*
Q: *Yes, she jumped five out of 4. And what other account could we do? What if we are going to make a sum? 5 x 4 is the same thing as adding up how many times the 4?*
C25: *What do you mean?*
Q: *She made 4, then she made 4 more, then 4 more, then 4 and 4 more. She did 4 four times, right? It was 20. That's right. If you were to do an extra count, how many times would you add the 4?*
C25: *Five.*
Q: *Yes, you would do 4 + 4 + 4 + 4 + 4 = 20 or 5 x 4, which is also equal to 20.*

Again, the discussion arose about the size of the jump and the number of jumps. To perform the multiplication, the order of the factors does not change the result. But, since there were four size 5 jumps, and not five size 4 jumps, it was important for students to understand this difference. C31 was the next to play. The student was thoughtful to make two jumps of length 5. The dialogue followed with the teacher's mediations, in the same way as the previous ones, showing the difference between two jumps of 5 and five jumps of 2.

This example that was reflected and explored by the teacher-researcher presents the distinction between the terms of multiplication. Although commutativity indicates that the order of factors does not change the product, it is from problem situations that each of the multiplication terms can be identified. In this case, number of hops and size of hops were different attributes.

Then, it was the turn of C24, who made two jumps of length 2.

Q: *There are two jumps of 2.* [With] *the first jump, where do you have to fall?*
C24: *On 2.*
Q: *That, and then what? There are two jumps of 2; you have so far made one.*
C24: *On 4.*
Q: *What account can you do to register?*
Q: *How many times have you skipped 2?*
C24: *Twice.*
Q: *How far?*
C24: *4.*
Q: *In total, it was 4, but how far was each jump?*
C24: *2.*
Q: *So, how is it? How many times did you skip size 2?*
C24: *Two.*
Q: *So, you jumped twice the distance of 2. So, you did 2 x 2, what is it?*
C24: *4.*
Q: *2 x 2 is 4. And the half of 4 is?*
C25: *2.*
Q: *To know half, what do I need to do?*
C24: *It is divided or times.*
C25: *It is divided, right?*
Q: *This, I divide by 2.*
The teacher went to the straight and positioned herself on top of the 2, asking.

Q: *If two is the half of 4, let's look: from four to 2, how much?*
Some: *2.*
Q: *And from zero to 2, how much?*
Some: *2.*
Q: *Is* [the measurement] *the same on both sides?*
Some: *yes!*
Q: *There are 2: it is half of 4. Half is when I divide it into two equal parts.*
Q: *2 x 2 is 4, and 4 ÷ 2 is 2.*

In this dialogue, the teacher-researcher problematized, once again, the term half to work on the concept of division involved in the multiplicative field. He used the line to indicate the half, exact division by 2, and the game made it possible to visualize that term. Through the numerical line, it was possible to indicate that from 0 to 2 the distance is 2, and from 2 to 4 the distance is also 2. In this way, it is easier for the teacher to explain and for the students to understand, from the that lead them to memorize, without understanding, the reverse operations. The teacher-researcher realized that this way is more interesting for the students, than teaching them to seek the result of the exact division in the multiplication table, in a mechanical way.

In this way, it is possible to develop the numerical field in children's learning, expanding the understanding of the concepts and properties of the Decimal Numbering System, while the teacher-researcher attests to her professional development from the reflection on other pedagogical practices.

C10 played, in sequence, and performed five jumps of length 3. Initially, the student jumped without respecting the size of the jump.

Q: *There are five jumps of 3. The first jump will fall on ...*
C10: *3.*
Q: *And what next?*
C10: *6.*
The student jumped 3 by 3, until she reached 15.
Q: *How many times did she skip 3?*
C19: *Five.*
Q: *How is it?*
C19: *5 x 3.*
Q: *Yes, five 3, 5 x 3 hops.*
C3 [who was helping to roll the dice]: *Or it could be 3 + 3 + 3 + 3 + 3 = 15.*

C3 made the relationship between different representations. As well as 5 x 3 = 15, it is also possible to do 3 + 3 + 3 + 3 + 3 = 15. Soon after, the C19 performed five jumps of size 2, quickly, until reaching 10, and then returned 2 by 2, until it reaches zero.

Q: *Back in 10. You did 5 x 2 and you scored 10. And if I ask you to do 10 ÷ 2, how much will it be?*
C19: *Give 5.*
C3: *It's half.*
Q: *So, let's check if 5 is really half of 10.*
The teacher asked C19 to stay at 5, C3 to stay at 10, and C7 at zero.
Q: *Let's check it out.*

C3: *It is because 5 plus 5 gives 10.*
Q: *Let's check the line. From C7 to C19, how much?*
Some: *Five.*
Q: *And from C19 to C3? How much?*
Some: *Five.*
Q: *So, really, half of 10 gives 5. Half is dividing by 2; 10 ÷ 2 gave 5 for each side. So, 5 x 2 = 10 and 10 ÷ 2 = 5.*

This dialogue contributed to the understanding of the teacher-researcher's actions on multiplication and its inverse - division -, which contributed to the work with games. In the line "*I asked C19 to stay at 5, C3 to stay at 10, and C7 at zero*", it is possible to notice that the teacher was concerned with explaining the concept of half, showing the equal quantities on each side of the line. She asked the students to stay on top of the numbers to show that, from zero to 5 and from 5 to 10, the size of the line is equal, and that is why there was, in the line, represented half of 10, two equal parts.

Final considerations

The article presents an excerpt from the investigation, presenting the analysis of some episodes of the use of the game "A boot of many leagues". It contributed so that children could better understand the multiplicative field, with an emphasis on the perception of the attribution of meaning for each factor of multiplication. The reflections of the teacher-researcher indicated the importance of the game as a viable resource for the exploration of multiplication, in aspects such as teaching the concept of half and double, when two variables are involved.

Even provoking the understanding of commutativity, the teacher-researcher highlighted the fact that, in a problem situation, each factor has its own meaning. Reflecting, mediating and presenting new situations from the experience, collaborated for the students' learning, but also for the professional development of the teacher, who was improving her ways of teaching, incorporating some resources and producing others, which promoted the teaching of concepts that had difficulty explaining.

The use of the line allowed the visualization of the term half, which, many times, is only transmitted through the use of a rule and merely decorated by the students, without being fully understood. The dialogues demonstrated that the mediations of the teacher-researcher in the use of the game allowed a process of signifying the concepts involved.

The practice also contributed to the understanding of the concept of division, new for most of the students, because the "sharing account" had not been addressed in that class as the inverse of multiplication. It also provided work with concepts already learned, such as the multiplication of numbers 1, 2, 3, 4 and 5, in addition to contributing to understanding by those who had difficulties in mathematics.

After the plays, the students recorded the jumps on printed lines, one for each child. The number of jumps on the line was easily recorded. However, in the calculation record, some inverted the multiplicand and the multiplier, as occurred in some of the plays, demonstrating that, for some, it is still difficult to understand which number is being multiplied, the commutation of multiplication prevailing. Although this guarantees the correctness of the result, it does not guarantee the understanding of the situation presented.

This process led the teacher-researcher to reflect that, perhaps, this occurs because the school practice is guided, sometimes, more in the valuation of the correct answer than in the process of understanding the problem situation presented. The answer may be correct, but that shouldn't be the most important thing. The student needs to understand the problem situation, which leads us to reflect that the order of factors does not change the result, but may not guarantee the understanding process, nor the understanding of the problem situation presented, which is the most important in the mathematics learning process.

We affirm, from the above, that the learning of mathematics should not happen through the use of mechanizations, but, rather, from activities that can be meaningful to the student and promote advances in mathematical thinking, especially in the early years. , when the mathematical literacy process is established. Therefore, emphasizing only the right answers is not the ideal way to assign meaning to activities.

It is also worth noting that the role of the teacher-researcher to create an environment conducive to an activity is paramount, especially with regard to stimulating communication in mathematics classes, respecting and listening to students' arguments. Promoting communication between students and mediation between teacher and children is a path that aims to understand mathematical thinking, without limiting itself to emphasizing mechanical processes that guarantee correct answers. Such statements were experienced in the course of the research.

The experiences lived by the teacher-researcher allowed her to develop her reflective processes, to give visibility to what was lived in the classroom, contributing significantly to her education and also with other mathematics teachers in the early years. The research of the practice itself emerges from real situations, which were and are experienced by the researcher teacher who explores alternatives, constantly seeking to improve his practice, in favor of pedagogical actions that contribute to the children's learning.

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