COMBINATORIAL TECHNIQUE FOR OPTIMIZING THE COMBINATION

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\textbf{A B S T R A C T}

This paper presents an innovative computing method and models for optimizing the combination defined in combinatorics. The optimized combination has been derived from the iterative computation of multiple geometric series and summability by specialized approach. The optimized combinatorial technique has applications in science, engineering and management. In this paper, several properties and consequences on the innovative optimized combination has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today’s challenges.

\textbf{k e y w o r d s}
Optimized Combination
Combinatorics
Counting technique
Binomial coefficient
1. INTRODUCTION

Combinatorics is a collection of various counting techniques or methods and models and has many applications in science, technology, and management. In the research paper, optimized combination of combinatorics is introduced that is useful for scientific researchers who are solving scientific problems and meeting today’s challenges.

2. OPTIMIZED COMBINATION

This optimized combination is derived from the iterative computations [Annamalai et al., 2018, 2019, 2020] of multi-geometric series and summability as follows

\[ \sum_{i=0}^{n-1} \sum_{i_2=1}^{n-1} \cdots \sum_{i_n=n-1}^{n-1} x^{i_1} = \sum_{i=0}^{n-1} V_i^{p} x^i, \quad (p \in \mathbb{N} \& 1 \leq p \leq n - 1) \]  

(A)

Where \( V_i^p \) is a binomial coefficient and its mathematical expressions are given below:

\[ V_i^p = \frac{(i + 1)(i + 2)(i + 3) \cdots (i + p)}{p!} \quad (1 \leq p \leq n - 1). \]

\[ V_{i-k}^{p} = \frac{(i - k + 1)(i - k + 2)(i - k + 3) \cdots (i - k + p)}{p!} \]

Let us prove the equation (A) using the multiple geometric series.

\[ \sum_{i_1=0}^{n-1} \sum_{i_2=1}^{n-1} x^{i_1} = \sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n-1} x^{i_2} = \sum_{i=0}^{n-1} \frac{(i + 1)}{1!} x^i = \sum_{i=0}^{n-1} V_i^1 x^i \]

Where

\[ \sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n-1} x^{i_2} = 1 + 2x + 3x^2 + \cdots + nx^{n-1}. \]

The growing complexity of mathematical modelling and its application demands the simplicity of numerical equations and combinatorial techniques for solving the scientific problems facing today. In view of this idea, the optimized combination of combinatorics is introduced that is

\[ V_i^n = \frac{(r + 1)(r + 2)(r + 3) \cdots (r + n - 1)(r + n)}{n!}, \]

where \( N = \{0, 1, 2, 3, 4, 5, \ldots \} \) is the set of natural numbers including the element 0.

If we continue like this, the binomial coefficient of the multiseries is \( V_i^p(1 \leq p \leq n - 1). \)
To convert the combination \( nCr \) into the optimized combination:

\[
nCr = \frac{n!}{r!(n-r)!} = (V_r^n)(V_{n-r}^r) \quad \text{where} \quad V_0^r = 1.
\]

Let us consider \( n - r = k \) for easily understood.

Then, 

\[
V_r^n = V_r^k = \frac{(r + 1)(r + 2)(r + 3) \cdots (r + k)}{k!}.
\]

To convert the combination \( nCn \) into the optimized combination:

\[
nCn = \frac{n!}{n!} = V_0^n = 1.
\]

To convert the combination \((n + r)Cr\) into the optimized combination:

\[
(n + r)Cr = \frac{n!}{r!(n + r - r)!} = \frac{n!}{r! n!} = \frac{1.2.3 \cdots r(r + 1)(r + 2) \cdots (r + n)}{r! n!} = (V_r^n)(V_r^n).
\]

Now \( V_r^n \) \((n, r \in N, n \geq 1, & r \geq 0)\) is considered as optimized combination.

Some results with proofs on the optimized combination [Annamalai, 2020] are provided below.

**Result 1:** \( V_0^1 = V_0^n = 1 \)

Proof. \( V_0^1 = \frac{(0 + 1)}{1!} = 1 \) \hspace{1cm} (i)

\[
V_0^n = \frac{(0 + 1)(0 + 2)(0 + 3) \cdots (0 + n)}{n!} = \frac{n!}{n!} = 1
\] \hspace{1cm} (ii)

From (i) and (ii), the result 1 is true.

**Result 2:** \( V_r^{n+1} - V_r^n = V_{r-1}^n \)

Proof. \( V_r^n = \frac{(r + 1)(r + 2) \cdots (r + n)}{n!} \)

\[
V_r^{n+1} = \frac{(r + 1)(r + 2) \cdots (r + n)(r + n + 1)}{(n + 1)!}
\]

\[
V_r^{n+1} - V_r^n = \frac{(r + 1)(r + 2) \cdots (r + n)(r + n + 1)}{(n + 1)!} - \frac{(r + 1)(r + 2) \cdots (r + n)}{n!} \cdot \frac{r + n + 1}{n + 1} - 1
\] \hspace{1cm} (iii)

It is understood from (iii) that the result 2 is true.

**Result 3:** \( 1 + V_1^1 + V_2^1 + V_3^1 \cdots V_1^n = V_2^n \)

Proof. \( V_2^n = \frac{(2 + 1)(2 + 2)(2 + 3) \cdots (2 + n - 1)(2 + n)}{n!} = \frac{(n + 1)(n + 2)}{2!} \) \hspace{1cm} (iv)

\[
1 + V_1^1 + V_1^2 + V_1^3 \cdots V_1^n = 1 + 2 + 3 + \cdots + n + 1 = \frac{(n + 1)(n + 2)}{2!}
\] \hspace{1cm} (v)

From (iv) and (v), the result 3 is true.

**Result 4:** \( V_r^n = V^n_r \) \((n, r \geq 1 \& n, r \in N)\)

Proof. \( V^n_r \) implies \( \frac{(r + 1)(r + 2) \cdots (r + n)}{n!} = \frac{(n + 1)(n + 2) \cdots (n + r)}{r!} \)
Assume that \( r = n + m \) (\( m \in N \& m \geq 1 \)). Let us show that \( V_{n+m}^n = V_{n}^{n+m} \).

\[
V_{n+m}^n = \frac{(n+m+1)(n+m+2) \cdots (n+m+n)}{n!} = \frac{(n+1)(n+2) \cdots (n+m+n)}{(n+m)!} \quad \text{(vi)}
\]

\[
V_{n+m}^n = \frac{(n+1)(n+2) \cdots (n+n) \cdots (n+n+1)(n+n+2) \cdots (n+n+m)}{(n+m)!} \quad \text{(vii)}
\]

From (vi) and (vii), \( V_{n+m}^n = V_{n}^{n+m} \) is true.

Assume that \( r = n - m \) (\( n > m \)). Let us show that \( V_{n-m}^n = V_{n}^{n-m} \).

\[
V_{n-m}^n = \frac{(n-m+1)(n-m+2) \cdots (n-m+n)}{n!} = \frac{(n+1)(n+2) \cdots (n+n-m)}{(n-m)!} \quad \text{(viii)}
\]

\[
V_{n-m}^n = \frac{(n+1)(n+2) \cdots (n+n-m)}{(n-m)!} \quad \text{(ix)}
\]

From (viii) and (ix), \( V_{n-m}^n = V_{n}^{n-m} \) is true.

If \( r = n \), \( V_{r}^n \) is obviously true for \( r = n \).

Hence, the result 4 is true.

**Result 5:** \( V_{n}^n = 2V_{n-1}^n \)

Proof. \( V_{n}^n = \frac{(n+1)(n+2) \cdots (n+n-1) \cdot 2n}{(n-1)! \cdot n} = \frac{2(n+1)(n+2) \cdots (n+n-1)}{(n-1)!} = 2V_{n-1}^n \)

Hence, the result 5 is true.

**Result 6:** \( V_{0}^n + V_{1}^n + V_{2}^n + V_{3}^n + \cdots + V_{r-1}^n + V_{r}^n = V_{r+1}^n \)

Proof. This result is proved by mathematical induction.

**Basis.** Let \( r = 1 \). \( V_{0}^n + V_{1}^n = V_{1}^{n+1} \) implies \( n+2 = n+2 \).

**Inductive hypothesis.**

Let us assume that \( V_{0}^n + V_{1}^n + V_{2}^n + \cdots + V_{k-1}^n = V_{k}^{n+1} \) is true for \( r = k - 1 \).

**Inductive step.** We must show that the inductive hypothesis is true for \( r = k \).

\( V_{0}^n + V_{1}^n + \cdots + V_{k}^n + V_{k}^n = V_{k}^{n+1} \) implies \( V_{0}^n + V_{1}^n + \cdots + V_{k}^n + V_{k}^n = V_{k}^{n+1} = V_{n}^{n+1} \).

Hence, it is proved.

### 3. CONCLUSION

In the research paper, optimized combination of combinatorics has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today’s challenges. The optimized combination was derived from the recursive computation of multigeometric series and summability. The combinatorial computing technique, called optimized combination, has been applied in differential and integral equation developed by using multiple geometric series and summability.

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