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COMBINATORIAL TECHNIQUE FOR OPTIMIZING THE COMBINATION

Chinnaraji Annamalai[1,*], Junzo Watada[2], Said Broumi[3] and Vishnu Narayan Mishra[4]

[1]Department of Management, Indian Institute of Technology,
Kharagpur, Paschim Medinipur, West Bengal – 721302, India
ORCID: https://orcid.org/0000-0002-0992-2584
[2]Graduate School of Information, Production and Systems, Waseda University,
1-104 Totsukamachi, Shinjuku-ku, Tokyo, 69-8050, Japan
ORCID: https://orcid.org/0000-0002-3322-2086
[3]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II,
B.P 7955, Sidi Othman, Casablanca, Morocco
ORCID: https://orcid.org/0000-0002-1334-5759
[4]Department of Mathematics, Indira Gandhi National Tribal University,
Lalpur, Amarkantak, Anuppur, Madhya Pradesh, India
ORCID: https://orcid.org/0000-0002-2159-7710

*Corresponding author. Indian Institute of Technology, Kharagpur, Paschim Medinipur, West Bengal, India, Phone: +91 3222 282155
e-mail addressl: anna@iitkgp.ac.in (Chinnaraji Annamalai).*

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A B S T R A C T

This paper presents an innovative computing method and models for optimizing the combination defined in combinatorics. The optimized combination has been derived from the iterative computation of multiple geometric series and summability by specialized approach. The optimized combinatorial technique has applications in science, engineering and management. In this paper, several properties and consequences on the innovative optimized combination has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

1. INTRODUCTION

Combinatorics is a collection of various counting techniques or methods and models and has many applications in science, technology, and management. In the research paper, optimized combination of combinatorics is introduced that is useful for scientific researchers who are solving scientific problems and meeting today's challenges.

2. OPTIMIZED COMBINATION

The growing complexity of mathematical modelling and its application demands the simplicity of numerical equations and combinatorial techniques for solving the scientific problems facing today. In view of this idea, the optimized combination of combinatorics is introduced that is

$$V_r^n = \frac{(r+1)(r+2)(r+3)\cdots\cdots(r+n-1)(r+n)}{n!},$$
$$(n, r \in N, n \geq 1, \& \, r \geq 0)$$

where N = {0, 1, 2, 3, 4, 5, ... } is the set of natural numbers including the element 0.

This optimized combination is derived from the iterative computations [Annamalai et al., 2018, 2019, 2020] of multi-geometric series and summability as follows

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\cdots\cdots\sum_{i_n=i_{n-1}}^{n-1} x^{i_n} = \sum_{i=0}^{n-1} V_i^p x^i, \ (p \in N \ \& \ 1 \leq p \leq n-1) \qquad \text{(A)}$$

Where V_i^p is a binomial coefficient and its mathematical expressions are given below:

$$V_i^p = \frac{(i+1)(i+2)(i+3)\ldots(i+p)}{p!} \quad (1 \leq p \leq n-1).$$

$$V_{i-k}^p = \frac{(i-k+1)(i-k+2)(i-k+3)\ldots(i-k+p)}{p!}.$$

Let us prove the equation (A) using the multiple geometric series.

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1} x^{i_2} = \sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n} x^{i_2} = \sum_{i=0}^{n-1}\frac{(i+1)}{1!}x^i = \sum_{i=0}^{n-1} V_i^1 x^i$$

where $\sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n} x^{i_2} = 1 + 2x + 3x^2 + \cdots + \frac{n}{1!}x^{n-1}.$

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} = \sum_{i_2=0}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \sum_{i_2=1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \sum_{i_2=2}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \cdots + \sum_{i_2=n-1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3}$$
$$= (1 + 2x + 3x^2 + \cdots + nx^{n-1}) + (x + 2x^2 + 3x^3 \cdots + (n-1)x^{n-1}) + \cdots x^{n-1}$$
$$= 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{n(n+1)}{2!}x^{n-1} = \sum_{i=0}^{n-1}\frac{(i+1)(i+2)}{2!}x^i = \sum_{i=0}^{n-1} V_i^2 x^i$$

where $\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + \cdots + \frac{n(n+1)}{2!}x^{n-1}.$

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\sum_{i_4=i_3}^{n-1} x^{i_4} = \sum_{i=0}^{n-1}\frac{(i+1)(i+2)(x+3)}{3!}x^i = \sum_{i=0}^{n-1} V_i^3 x^i$$

where $\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\sum_{i_4=i_3}^{n-1} x^{i_4} = 1 + 4x + 10x^2 + 20x^3 + 35x^4 + \cdots + \frac{n(n+1)(n+2)}{3!}x^{n-1}.$

If we continue like this, the binomial coefficient of the multisereis is $V_i^p (1 \leq p \leq n-1)$.

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To convert the combination nCr into the optimized combination:

$nCr = \dfrac{n!}{r!\,(n-r)!} = (V_0^r)(V_r^{n-1}) = V_r^{n-r}$ where $V_0^r = 1$.

Let us consider $n - r = k$ for easily understood.

Then, $\quad V_r^{n-r} = V_r^k = \dfrac{(r+1)(r+2)(r+3)\cdots(r+k)}{k!}$.

To convert the combination nCn into the optimized combination:

nCn$=\frac{n!}{n!} = V_0^n = 1$.

To convert the combination $(n+r)Cr$ into the optimized combination:

$(n+r)Cr = \dfrac{n!}{r!\,(n+r-r)!} = \dfrac{n!}{r!\,n!} = \dfrac{1.2.3\cdots r(r+1)(r+2)\cdots(r+n)}{r!\,n!} = (V_0^r)(V_r^n)$.

$(V_0^r)(V_r^n) = V_r^n$ where $V_0^r = 1$.

Now V_r^n $(n, r \in N, n \geq 1, \& \, r \geq 0)$is considered as optimized combination.

Some results with proofs on the optimized combination [Annamalai, 2020] are provided below.

Result 1: $\boldsymbol{V_0^1 = V_0^n = 1}$

Proof. $V_0^1 = \dfrac{(0+1)}{1!} = 1$ (i)

$V_0^n = \dfrac{(0+1)(0+2)(0+3)\cdots\cdots(0+n)}{n!} = \dfrac{n!}{n!} = 1$ (ii)

From (i) and (ii), the result 1 is true.

Result 2: $\boldsymbol{V_r^{n+1} - V_r^n = V_{r-1}^n}$

Proof. $V_r^n = \dfrac{(r+1)(r+2)(r+3)\cdots\cdots(r+n)}{n!}$

$V_r^{n+1} = \dfrac{(r+1)(r+2)(r+3)\cdots\cdots(r+n)(r+n+1)}{(n+1)!}$

$V_r^{n+1} - V_r^n = \dfrac{(r+1)(r+2)(r+3)\cdots\cdots(r+n)}{n!}\left[\dfrac{r+n+1}{n+1} - 1\right]$

$V_r^{n+1} - V_r^n = \dfrac{r(r+1)(r+2(r+3)\cdots\cdots(r+n)}{n!} = V_{r-1}^n$ (iii)

It is understood from (iii) that the result 2 is true.

Result 3: $\boldsymbol{1 + V_1^1 + V_1^2 + V_1^3 \cdots\cdots V_1^n = V_2^n}$

Proof. $V_2^n = \dfrac{(2+1)(2+2)(2+3)\cdots\cdots(2+n-1)(2+n)}{n!} = \dfrac{(n+1)(n+2)}{2!}$ (iv)

$1 + V_1^1 + V_1^2 + V_1^3 \cdots\cdots V_1^n = 1 + 2 + 3 + \cdots + n + 1 = \dfrac{(n+1)(n+2)}{2!}$ (v)

From (iv) and (v), the result 3 is true.

Result 4: $\boldsymbol{V_r^n = V_n^r}$ $\boldsymbol{(n, r \geq 1}$ & $\boldsymbol{n, r \in N)}$

Proof. $V_r^n = V_n^r$ implies $\dfrac{(r+1)(r+2)\cdots(r+n)}{n!} = \dfrac{(n+1)(n+2)\cdots(n+r)}{r!}$

Assume that $r = n + m$ $(m \in N \text{ \& } m \geq 1)$. Let us show that $V_{n+m}^{n} = V_{n}^{n+m}$.

$$V_{n+m}^{n} = \frac{(n+m+1)(n+m+2)\cdots(n+m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+m+n)}{(n+m)!} \qquad \text{(vi)}$$

$$V_{n}^{n+m} = \frac{(n+1)(n+2)\cdots(n+n)(n+n+1)(n+n+2)\cdots(n+n+m)}{(n+m)!} \qquad \text{(vii)}$$

From (vi)and (vii), $\quad V_{n+m}^{n} = V_{n}^{n+m} \text{ is true}.$

Assume that $r = n - m$ $(n > m)$. Let us show that $V_{n-m}^{n} = V_{n}^{n-m}$.

$$V_{n-m}^{n} = \frac{(n-m+1)(n-m+2)\cdots(n-m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!} \qquad \text{(viii)}$$

$$V_{n}^{n-m} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!} \qquad \text{(ix)}$$

From (viii))and (ix), $V_{n-m}^{n} = V_{n}^{n-m}$ is true.

If $r = n$, $V_{r}^{n} = V_{n}^{r}$ is obivously true for $r = n$.

Hence, the result 4 is true.

Result 5: $\boldsymbol{V_{n}^{n} = 2V_{n-1}^{n}}$

Proof. $V_{n}^{n} = \dfrac{(n+1)(n+2)\cdots(n+n-1)2n}{(n-1)!\,n} = \dfrac{2(n+1)(n+2)\cdots(n+n-1)}{(n-1)!} = 2V_{n-1}^{n}$

Hence, the result 5 is true.

Result 6: $\boldsymbol{V_{0}^{n} + V_{1}^{n} + V_{2}^{n} + V_{3}^{n} \cdots + V_{r-1}^{n} + V_{r}^{n} = V_{r}^{n+1}}$

Proof. This result is proved by mathematical induction.

Basis. Let $r = 1$. $V_{0}^{n} + V_{1}^{n} = V_{1}^{n+1}$ implies $n + 2 = n + 2$.

Inductive hypothesis.

Let us assume that $V_{0}^{n} + V_{1}^{n} + V_{2}^{n} + \cdots + V_{k-1}^{n} = V_{k-1}^{n+1}$ is true for $r = k - 1$.

Inductive step. We must show that the inductive hypothesis is true for $r = k$.

$V_{0}^{n} + V_{1}^{n} + \cdots + V_{k-1}^{n} + V_{k}^{n} = V_{k}^{n+1}$ implies $V_{0}^{n} + V_{1}^{n} + \cdots + V_{k-1}^{n} = V_{k}^{n+1} - V_{k}^{n} = V_{k-1}^{n+1}$.

Hence, it is proved.

3. CONCLUSION

In the research paper, optimized combination of combinatorics has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges. The optimized combination was derived from the recursive computation of mult-geometric series and summability. The combinatorial computing technique, called optimized combination, has been applied in differential and integral equation developed by using multiple geometric series and summability.

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