Series and Summations on Binomial Coefficients of Optimized Combination

Article Info:
Article history: Received 2021-12-22 / Accepted 2022-03-20 / Available online 2022-04-19
doi: 10.18540/jcecvl8iss3pp14123-01

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Abstract
This paper presents innovative series and summations derived from the optimized combinations relating to the combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems.

Keywords: optimized combination, binomial coefficient, binomial series.

1. Introduction to Optimized Combination

The optimized combination (Annamalai et al., 2020) applied on the computation of multiple geometric series (Annamalai, 2010) is expressed as follows:

\[ V_r^n = \frac{(r+1)(r+2) \cdots (r+n)}{n!} = \frac{(n+1)(n+2) \cdots (n+r)}{r!} = V_n^r, \]

i. e., \[ V_r^n = \prod_{i=1}^{n} \frac{r+i}{n!} = \prod_{i=1}^{r} \frac{n+i}{r!} = V_n^r \quad (n, r \geq 1 \text{ and } n, r \in N), \]

where \( N = \{0, 1, 2, 3, \cdots\} \), \( V_r^n \) is a binomial coefficient, and \( n! \) is the factorial of \( n \).

Some results (Annamalai et al., 2020) of the optimized combination are provided below:

i). \[ V_n^0 = V_0^n = 1 \quad (n \geq 1 \text{ and } n \in N), \]
where $V_n^0$ alaway implies $V_n^0$, i.e., $V_n^0 \Rightarrow V_n^0$.

Note that $V_r^n = V_r^n = (n + r)C_r = (n + r)C_n = \frac{(n + r)!}{n! \cdot r!}$ and $V_0^0 = 1$.

ii). $V_r^n = V_r^n \ (n, r \geq 1 \& n, r \in N) \& V_0^n = V_0^n$.

iii): $V_0^n + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \Rightarrow \sum_{i=0}^{n} V_i^n = V_r^{n+1} \ (n, r \in N)$.

2. Novel Series of Optimized Combination

From the result (iii) in this paper, that is (Annamalai, 2020),

$$1 + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \iff 1 + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1},$$

the following series and its summations (Annamalai, 2018 & Annamalai, 2019) are expressed:

(1). $\sum_{i=0}^{n} \frac{(i + 1)}{1!} = 1 + 2 + 3 + \cdots + n + (n + 1) = \frac{(n + 1)(n + 2)}{2!}$.

(2). $\sum_{i=0}^{n} \frac{(i + 1)(i + 2)}{2!} = 1 + 3 + 6 + \cdots + \frac{(n + 1)(n + 2)}{2!} = \frac{(n + 1)(n + 2)(n + 3)}{3!}$.

(3). $\sum_{i=0}^{n} \frac{(i + 1)(i + 2)(i + 3)}{3!} = \frac{(n + 1)(n + 2)(n + 3)(n + 4)}{4!}$.

(4). $\sum_{i=0}^{n} \frac{(i + 1)(i + 2)(i + 3)(i + 4)}{4!} = \frac{(n + 1)(n + 2)(n + 3)(n + 4)(n + 5)}{5!}$.

Similarly, the series continues upto $r$ times. The $r^{th}$ series and its summation are:

(5). $\sum_{i=0}^{n} \frac{(i + 1)(i + 2)(i + 3) \cdots (i + r)}{r!} = \frac{(n + 1)(n + 2) \cdots (n + r)(n + r + 1)}{(r + 1)!}$.

i.e., $\sum_{i=0}^{n} \prod_{j=1}^{r} \frac{i + j}{r!} = \prod_{i=1}^{r+1} \frac{n + i}{(r + 1)!}$.

3. Conclusion

In this paper, the innovative series and summations of binomial coefficients have been derived using the results (Annamalai, 2022) of the optimized combination in the field of combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems and meet today’s challenges (Annamalai, 2010).
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