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Abelian Group on the Binomial Coefficients in Combinatorial Geometric Series

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Abstract
This paper discusses an abelian group, also called a commutative group, under addition of the binomial coefficients of combinatorial geometric series. The combinatorial geometric series is derived from the multiple summations of geometric series The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. This idea can enable the scientific researchers to solve the real life problems.
Keywords: computation, combinatorics, binomial coefficient, abelian group

1. Introduction
When the author of this article was trying to compute the multiple summations of geometric series (Annamalai, 2010, 2017a, 2017b, 2017c, 2018a, 2018b, 2018c, 2018d, 2019a, 2019b, 2020), a new idea stimulated his mind to create a combinatorial geometric series (Annamalai, 2022a, 2022b, 2022c). The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this paper, a commutative group, also called an abelian group, under addition of the binomial coefficients of combinatorial geometric series (Annamalai, 2022d, 2022e) is introduced newly.

2. Combinatorial Geometric Series
The combinatorial geometric series (Annamalai, 2022d, 2022e, 2002h) is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial geometric series refers to the binomial coefficient V_n^r (Annamalai, 2022i, 2022k).

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \;\;\&\;\; V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1 \;\; and \;\; n, r \in N = \{1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r *is* the binomial coefficient for combinatorial geometric series.

$V_0^1 = 1;\; V_1^1 = 2;\;\; V_2^1 = 3;\; V_3^1 = 4;\; V_4^1 = 5;\; V_5^1 = 6;\; \cdots$

$N = \{V_0^1, V_1^1, V_2^1, V_3^1,\ V_4^1, V_4^1, \cdots\}$ is a set of natirual umbers .

$W = \{0, V_0^1, V_1^1, V_2^1, V_3^1,\ V_4^1, V_4^1, \cdots\}$ is a set of whole umbers .

$Z = \{\cdots, -V_2^1, -V_1^1, -V_0^1, 0, V_0^1, V_1^1, V_2^1, \cdots\}$ is a set of integers.

$\{+, -, \times, \div, \cdots\}$ is a set of binary operators, where the symbol + is used for addition, the symbol – for subtraction, the symbol × for multiplication, the symbol ÷ for division, etc.

Note that $V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1} \ \Rightarrow V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$,
where $V_n^r = V_0^{r-1} + V_1^{r-1} + V_2^{r-1} + \cdots + V_n^{r-1}$ and $V_0^r + V_1^r + V_2^r + \cdots + V_{n-1}^r = V_{n-1}^{r+1}$.

Proof for $V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1}$.

$V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1} \Rightarrow V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$.

Note that $V_n^r = \dfrac{(n+r)!}{n!\,r!}$.

$$V_n^r + V_{n-1}^{r+1} = \frac{(n+r)!}{n!\,r!} + \frac{(n-1+r+1)!}{(n-1)!\,(r+1)!} = (n+r)!\left(\frac{r+1}{n!\,(r+1)!} + \frac{n}{n!\,(r+1)!}\right).$$

$$V_n^r + V_{n-1}^{r+1} = (n+r)!\left(\frac{n+r+1}{n!\,(r+1)!}\right) = \frac{(n+r+1)!}{n!\,(n+r)!} = V_n^{r+1}.$$

$\therefore V_n^r + V_{n-1}^{r+1} = V_n^{r+1}$.

Hence, it is proved.

Also, $V_n^r = V_n^{r-1} + V_r^{n-1}$, which is the sum of partitions of V_n^r.

In general, addition or multiplication of any two binomial coefficients is an integer as all binomial coefficients are integers such that $V_n^r \in N$.

1. Abelian Group

$Z = \{-V_r^n, 0, V_r^n \mid n \geq 1, r \geq 0\ \ \&\ \ n, r \in N\}$ is a set of integers.

Closure property: Addition of any two binomial coefficients is also a binomial coefficient.
$\left(V_m^n + V_p^q\right) \in Z$ for all $V_m^n, V_p^q \in Z$.

Associativity: For all $V_m^n, V_p^q, V_r^s \in Z,\ V_m^n + (V_p^q + V_r^s) = (V_m^n + V_p^q) + V_r^s$.

Identity element: $0 + V_r^n = V_r^n + 0 = V_r^n$, where 0 is an identity element.

Inverse element: $V_r^n + (-V_r^n) = (-V_r^n) + V_r^n = 0$, where $-V_r^n$ is an additive inverse.

Commutativity: $V_m^n + V_p^q = V_p^q + V_m^n$ for $V_m^n, V_p^q \in Z$.

(Z, +) is an abelian group under addition.

3. Conclusion

In this article, an abelian group was formed on the binomial coefficients of combinatorial geometric series under addition. This new idea can enable the scientific researchers to solve the real life problems (Annamalai, 2010).

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