

## Skew Field on the Binomial Coefficients in Combinatorial Geometric Series

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### Abstract

This paper discusses a commutative group, ring, and field under addition and multiplication of the binomial coefficients in combinatorial geometric series. The combinatorial geometric series is derived from the multiple summations of geometric series. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. This idea can enable the scientific researchers to solve the real life problems.

**Keywords:** Computation, Binomial Coefficient, Skew Field.

### 1. Introduction

When the author of this article was trying to compute the multiple summations of geometric series (Annamalai, 2010; Annamalai, 2017a; Annamalai, 2017b; Annamalai, 2017c; Annamalai, 2018a; Annamalai, 2018b; Annamalai, 2018c; Annamalai, 2018d; Annamalai, 2019a; Annamalai, 2019b; and Annamalai, 2020), a new idea stimulated his mind to create a combinatorial geometric series (Annamalai, 2022a; Annamalai, 2022b; Annamalai, 2022c; Annamalai, 2022d; Annamalai, 2022e; Annamalai, 2022f). The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient  $V_n^r$ . In this paper, a commutative group, ring, and field under addition of the binomial coefficients of combinatorial geometric series (Annamalai, 2022d; and Annamalai, 2022e) are introduced in detail.

### 2. Combinatorial Geometric Series

The combinatorial geometric series (Annamalai, 2022f; Annamalai, 2022g; Annamalai, 2002h) is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial geometric series refers to the binomial coefficient  $V_n^r$  (Annamalai, 2022i; Annamalai, 2022j; Annamalai & Siqueira, 2022b).

$$\sum_{i_1=0}^n \sum_{i_2=i_1}^n \sum_{i_3=i_2}^n \cdots \sum_{i_r=i_{r-1}}^n x^{i_r} = \sum_{i=0}^n V_i^r x^i \quad \& \quad V_n^r = \frac{(n+1)(n+2)(n+3) \cdots (n+r-1)(n+r)}{r!},$$

where  $n \geq 0, r \geq 1$  and  $n, r \in N = \{1, 2, 3, \dots\}$ .

Here,  $\sum_{i=0}^n V_i^r x^i$  refers to the combinatorial geometric series and

$V_n^r$  is the binomial coefficient for combinatorial geometric series.

$$V_0^1 = 1; V_1^1 = 2; V_2^1 = 3; V_3^1 = 4; V_4^1 = 5; V_5^1 = 6; \dots$$

$N = \{V_0^1, V_1^1, V_2^1, V_3^1, V_4^1, V_4^1, \dots\}$  is a set of natural numbers (Annamalai & Siqueira, 2022a).

$W = \{0, V_0^1, V_1^1, V_2^1, V_3^1, V_4^1, V_4^1, \dots\}$  is a set of whole numbers (Annamalai & Siqueira, 2022a).

$Z = \{\dots, -V_2^1, -V_1^1, -V_0^1, 0, V_0^1, V_1^1, V_2^1, \dots\}$  is a set of integers.

$\{+, -, \times, \div, \dots\}$  is a set of binary operators, where the symbol + is used for addition, the symbol - for subtraction, the symbol  $\times$  for multiplication, the symbol  $\div$  for division, etc.

**Theorem 4.4:**  $V_r^n - V_r^{n-r} = V_n^r - V_{n-r}^r, (n \geq r \ \& \ n, r \in N)$ .

$$\begin{aligned} \text{Proof. } V_r^n - V_r^{n-r} &= \frac{(r+1)(r+2)\dots(r+n-r)(r+n-r+1)\dots(r+n-r+r)}{(r+1)(r+2)(n-3)\dots(r+n-r)} \\ &\quad \frac{n!}{(n-r)!} \\ &= \frac{(r+1)(r+2)(n-3)\dots(r+n-r)}{(n-r)!} \left( \frac{(n+1)(n+2)\dots(n+r-1)(n+r)}{(n-r+1)(n-r+2)\dots(n-r+r)} - 1 \right). \end{aligned}$$

Here,  $r! V_n^r = (n+1)(n+2)\dots(n+r)$  and  $r! V_{n-r}^r = (n-r+1)(n-r+2)\dots(n-r+r)$ .

$$\therefore V_r^n - V_r^{n-r} = V_r^{n-1} \left( \frac{r! V_n^r}{r! V_{n-r}^r} - 1 \right) = V_r^{n-1} \left( \frac{V_n^r}{V_{n-r}^r} - 1 \right) = V_n^r - V_{n-r}^r.$$

Hence, theorem is proved.

### 3. Ring and Field

$Z = \{-V_r^n, 0, V_r^n \mid n \geq 1, r \geq 0 \ \& \ n, r \in N\}$  is a set of integers.

Closure property: Addition of any two binomial coefficients is also a binomial coefficient.

$(V_m^n + V_p^q) \in Z$  for all  $V_m^n, V_p^q \in Z$ .

Associativity: For all  $V_m^n, V_p^q, V_r^s \in Z, V_m^n + (V_p^q + V_r^s) = (V_m^n + V_p^q) + V_r^s$ .

Identity element:  $0 + V_r^n = V_r^n + 0 = V_r^n$ , where 0 is an additive identity.

Inverse element:  $V_r^n + (-V_r^n) = (-V_r^n) + V_r^n = 0$ , where  $-V_r^n$  is an additive inverse.

Commutativity:  $V_m^n + V_p^q = V_p^q + V_m^n$  for all  $V_m^n, V_p^q \in Z$ .

$(Z, +)$  is a commutative group under addition (Annamalai & Siqueira, 2022a).

A **RING** is a non-empty set R which is CLOSED under two binary operators + and  $\times$  and satisfying the following properties:

- (1) R is a commutative group under +.
- (2) R is an associativity of  $\times$ . For a, b, c  $\in$  R,  $a \times (b \times c) = (a \times b) \times c$ .
- (3) R has distributive properties, i.e. for all a, b, c  $\in$  R the following identities hold:  
 $a \times (b + c) = (a \times b) + (a \times c)$  and  $(b + c) \times a = (b \times a) + (c \times a)$ .

$\therefore (Z, +, \times)$  is a **RING**.

Note that  $(Z, +, \times)$  is a **Ring with Unity** which has 1 as multiplicative identity such that  $1 \times V_r^n = V_r^n \times 1 = V_r^n$  and also Commutative Ring:  $V_m^n \times V_p^q = V_p^q \times V_m^n$ .

A **FIELD** is a non-empty set F which is CLOSED under two binary operators + and  $\times$  and satisfying the following properties:

- (1) F is an abelian group under +.
- (2)  $F - \{0\}$  is an abelian group under  $\times$ .

$\therefore (Z, +, \times)$  is a **FIELD**.

Note. A division ring is a ring in which  $0 \neq 1$  and every nonzero element has a multiplicative inverse. A noncommutative division ring is called a skew field. A commutative division ring is called a field.

#### 4. Conclusion

In this article, a commutative group, ring, and field were formed on the binomial coefficients of combinatorial geometric series under addition and multiplication. This new idea can enable the scientific researchers for research and development further (Annamalai, 2010).

#### References

- Annamalai, C. (2010) Applications of exponential decay and geometric series in effective medicine dosage. *Advances in Bioscience and Biotechnology*, 1(1), 51-54. <https://doi.org/10.4236/abb.2010.11008>.
- Annamalai, C. (2017a) Analysis and Modelling of Annamalai Computing Geometric Series and Summability. *Mathematical Journal of Interdisciplinary Sciences*, 6(1), 11-15. <https://doi.org/10.15415/mjis.2017.61002>.
- Annamalai, C. (2017b) Annamalai Computing Method for Formation of Geometric Series using in Science and Technology. *International Journal for Science and Advance Research In Technology*, 3(8), 187-289. <http://ijsart.com/Home/IssueDetail/17257>.
- Annamalai, C. (2017c) Computational modelling for the formation of geometric series using Annamalai computing method. *Jñānābha*, 47(2), 327-330. <https://zbmath.org/?q=an%3A1391.65005>.
- Annamalai, C. (2018a) Algorithmic Computation of Annamalai's Geometric Series and Summability. *Journal of Mathematics and Computer Science*, 3(5), 100-101. <https://doi.org/10.11648/j.mcs.20180305.11>.
- Annamalai, C. (2018b) Annamalai's Computing Model for Algorithmic Geometric Series and Its Mathematical Structures. *Journal of Mathematics and Computer Science*, 3(1), 1-6. <https://doi.org/10.11648/j.mcs.20180301.11>.
- Annamalai, C. (2018c) Computing for Development of a New Summability on Multiple Geometric Series. *International Journal of Mathematics, Game Theory and Algebra*, 27(4), 511-513.
- Annamalai, C. (2018d) Novel Computation of Algorithmic Geometric Series and Summability. *Journal of Algorithms and Computation*, 50(1), 151-153. <https://www.doi.org/10.22059/JAC.2018.68866>.
- Annamalai, C. (2019a) A Model of Iterative Computations for Recursive Summability. *Tamsui Oxford Journal of Information and Mathematical Sciences*, 32(1), 75-77.
- Annamalai, C. (2019b) Recursive Computations and Differential and Integral Equations for Summability of Binomial Coefficients with Combinatorial Expressions. *International Journal of Scientific Research in Mechanical and Materials Engineering*, 4(1), 6-10. <https://ijsrmme.com/IJSRMME19362>.
- Annamalai, C. (2020) Combinatorial Technique for Optimizing the combination. *The Journal of Engineering and Exact Sciences*, 6(2), 0189-0192. <https://doi.org/10.18540/jcecvl6iss2pp0189-0192>.
- Annamalai, C. (2022a) Computation and Calculus for Combinatorial Geometric Series and Binomial Identities and Expansions. *The Journal of Engineering and Exact Sciences*, 8(7), 14648–01i. <https://doi.org/10.18540/jcecvl8iss7pp14648-01i>.
- Annamalai, C. (2022b) Application of Factorial and Binomial identities in Information, Cybersecurity and Machine Learning. *International Journal of Advanced Networking and Applications*, 14(1), 5258-5260. <https://doi.org/10.33774/coe-2022-pnx53-v21>.

- Annamalai, C. (2022c) Combinatorial and Multinomial Coefficients and its Computing Techniques for Machine Learning and Cybersecurity. *The Journal of Engineering and Exact Sciences*, 8(8), 14713–01i. <https://doi.org/10.18540/jcecvl8iss8pp14713-01i>.
- Annamalai, C. (2022d) Annamalai's Binomial Identity and Theorem, *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4097907>.
- Annamalai, C. (2022e) Computing Method for Combinatorial Geometric Series and Binomial Expansion. *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4168016>.
- Annamalai, C. (2022f) Computation of Multinomial and Factorial Theorems for Cryptography and Machine Learning. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks-v9>.
- Annamalai, C. (2022g) Computation of Binomial, Factorial and Multinomial Theorems for Machine Learning and Cybersecurity. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks-v11>.
- Annamalai, C. (2022h) Series and Summations on Binomial Coefficients of Optimized Combination. *The Journal of Engineering and Exact Sciences*, 8(3), 14123-01e. <https://doi.org/10.18540/jcecvl8iss3pp14123-01e>.
- Annamalai, C. (2022i) Factorials and Integers for Applications in Computing and Cryptography. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks>.
- Annamalai, C. (2022j) Multinomial Theorem on the Binomial Coefficients for Combinatorial Geometric Series. *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4202632>.
- Annamalai, C., & Siqueira, A. M. de O. (2022a). Abelian Group on the Binomial Coefficients in Combinatorial Geometric Series. *The Journal of Engineering and Exact Sciences*, 8(10), 14799–01i. <https://doi.org/10.18540/jcecvl8iss10pp14799-01i>
- Annamalai, C., & Siqueira, A. M. de O. (2022b). Lemma on the Binomial Coefficients of Combinatorial Geometric Series. *The Journal of Engineering and Exact Sciences*, 8(9), 14760–01i. <https://doi.org/10.18540/jcecvl8iss9pp14760-01i>