

Experimental Simulation of a Tennis Ball using Wind Tunnel

Simulação Experimental de uma Bola de Tênis utilizando um Túnel de Vento

Article Info:

Article history: Received 2022-11-22 / Accepted 2023-01-01 / Available online 2023-01-01 doi: 10.18540/jcecvl9iss1pp15179-01e



Cesar Almiro de Souza Federal University of Viçosa, Brazil E-mail:cesar.souza@ufv.br Julio Cesar Costa Campos ORCID: https://orcid.org/0000-0002-9488-8164 Federal University of Vicosa, Brazil E-mail: julio.campos@ufv.br Antonio Marcos de Oliveira Siqueira ORCID: https://orcid.org/0000-0001-9334-0394 Federal University of Viçosa, Brazil E-mail: antonio.siqueira@ufv.br Pedro Casanova Treto ORCID: https://orcid.org/0000-0001-8508-6293 Universidad de Costa Rica, Costa Rica E-mail: pedro.casanova@ucr.ac.cr Alvaro Messias Bigonha Tibiriça ORCID: https://orcid.org/0000-0002-3300-1988 Federal University of Vicosa, Brazil E-mail: alvaro.tibirica@ufv.br Henrique Márcio Pereira Rosa ORCID: https://orcid.org/0000-0002-1437-2265 Federal University of Viçosa, Brazil E-mail: henrique.rosa@ufv.br **Rogério Fernandes Brito** ORCID: https://orcid.org/0000-0002-6833-7801 Federal University of Itajubá, Brazil E-mail: rogbrito@unifei.edu.br

Resumo

Neste estudo, as bolas de ténis foram analisadas experimentalmente através da utilização de túnel de vento com velocidade que varia de 1m/s a 14m/s, a qual representa uma variação no número Reynolds de 10.000 < Re < 60.000). O método utilizado foi a avaliação dos aspectos aerodinâmicos das bolas, incluindo a posição da costura e o grau de penugem, ou seja, com e sem penugem. Foi possível analisar o efeito do arrastamento sobre o diâmetro, na investigação da relação entre o coeficiente de arraste, C_D e o número de Reynolds, Re, para bolas novas e usadas. Os gráficos foram gerados utilizando o número de Reynolds e o Coeficiente de Arrasto, a fim de avaliar a dependência ou não destes parâmetros. Nas medições efetuadas, foram consideradas as bolas estáticas dentro do túnel de vento, desta forma, sem rotação. Por conseguinte, não são apresentadas discussões sobre a força Magnus. Os resultados obtidos, $C_D \approx 3$ a $C_D \approx 0,60$, foram consistentes com o intervalo do número Reynolds investigado. São esperados valores elevados para o coeficiente de arrasto, para o intervalo do número de Reynolds examinado. A posição da costura, de acordo com as literaturas

relacionadas, é desprezível para valores elevados de Reynolds, ou seja, superior a 50.000. Por outro lado, para valores baixos de Reynolds, pode representar uma diferença de até cerca de 9% para C_D . As bolas sem penugem mostraram uma forte influência da posição da costura, o que caracteriza a influência deste parâmetro. O efeito da penugem parecia ser responsável por cerca de 10% do arrasto total para valores baixos de Reynolds. A variação do diâmetro foi analisada isoladamente. **Palavras-chaves**: Aerodinâmica. Bola de tênis. Túnel de vento. Coeficiente de Arrasto.

Abstract

In this study, tennis balls were analyzed experimentally through the use of a wind tunnel with speed ranging from 1m/s to 14 m/s, which is a variation in the Reynolds number (10,000 < Re < 60,000). In this context, aerodynamic aspects of the balls were evaluated, including the position of the seam and the degree fuzz, i.e., with and without fuzz. It was possible to analyze the effect of drag on the diameter, in the investigation of the relationship between the drag coefficient (C_D) and the Reynolds number (Re) for new and used balls. Graphics were generated using the Reynolds number and the Drag Coefficient in order to assess the (non) dependency of these parameters. In the measurements performed, the static balls inside the wind tunnel were considered, i.e., without rotation. Therefore, no discussions about the Magnus force are presented. The results obtained, $C_D \approx 3$ to $C_D \approx 0.60$, were consistent for the range of the Reynolds number investigated. High values are expected for the drag coefficient, to the range of Reynolds number examined. The position of the seam, according to the related literatures, is negligible to high values of Reynolds, i.e., Re >50,000. On the other hand, for low values of Reynolds number, it can represent a difference of up to about 9% for the C_D. The balls without fluff showed the strongest influence of the position of the seam, which characterizes the influence of this parameter. The effect of fuzz seemed to be responsible for about 10% of the total drag for low values of the Reynolds number. The diameter variation was analyzed alone. Keywords: Aerodynamics. Tennis ball. Wind tunnel. Drag coefficient.

1. INTRODUCTION

In sports, many researchers have been studying the aerodynamic characteristics of balls used to the practice of soccer, golf and tennis to allow the improvement of these balls. The examples related to sports activities are used to teach the study in fluid mechanics with the aid of the wind tunnel. The wind tunnel is a useful device to exemplify the aerodynamic behavior of several classic examples, such as the one discussed in this article. In this context, several researchers stand out as a reference for studies in sports, among them the following stand out: Achenbach (1972), Achenbach (1974), Alam *et al.* (2019); Alam *et al.* (2012); Djamovski *et al.* (2012); Kozlov *et al.* (2015); Moria *et al.* (2011); Driscoll *et al.* (2016); and Asai & Kamemoto (2011).

There are several publications in which the trajectory of a ball was modeled and can be used as additional resources to explain the aerodynamic theory of the ball, among them the following stand out: (Bray and Kerwin, 2003 and Choppin, 2013). In some cases, the model was validated against experimental data (Goff and Carré, 2009).

Specifically, in tennis, researches were motivated by the observation that scoreserves ffected the results of tennis matches, since the ball moves at high speed and the opponent and the audience cannot follow the displacement of the ball. To decrease the speed of scoreserves, in 1990, the International Tennis Federation (ITF) decided to carry out researches for achieving a bigger ball, which would directly increase the drag. However, it was noted that the increase in the drag attributed to an increase in the diameter alone would not generate the desired effect (Haake *et al.*, 2000). Hence, the inclusion of studies related to other properties of the balls, such as fuzz, seam and rotation, is desirable, since a change in the rules of the game or in the characteristics of the racket would make the sport unpopular.

The present research considered the ball in a static position in the evaluation section of the wind tunnel, so that the effect of sustenance was considered. Otherwise, it should be inserted a study

on the parameters of the Magnus force, observed by the deviation of the trajectory of the ball from a straight path initially expected. This would result in a curve of a lateral deflection in the movement of the ball, besides changes in the sustenance, depending on the rotation of the ball. Thus, the Magnus force results from the change in the displacement points of the boundary layer by the Bernoulli Effect.

Therefore, the main goal of this research is to investigate the behavior of the aerodynamic properties of tennis balls related to the effects of fuzz, seam position in relation to the runoff and effect of the diameter. Considering the knowledge of these properties, new balls were compared with used balls, of the same brand, in order to understand the variation in their behavior throughout their useful life.

2. EXPERIMENTAL PROCEDURE AND EQUIPMENTS

2.1. Wind Tunnel

The research was based on an investigation of the drag force for low Reynolds values. A wind tunnel of a test section was used to identify the runoff variables of the balls, as shown in Fig. 1, 190 mm by 170 mm with 200 mm long, and a maximal average speed of 14 m/s, i.e., 50.4 km/h and $Re_{max} = 5.8 \cdot 10^4$.



Figure 1. Representation of the wind tunnel

Source: Authors (2023).

Nine values of speed were taken, ranging from each other in about 1.5 m/s (5.4 km/h) in this interval. The analyses were carried out with static ball, within the test section, i.e., without analyzing the effect of rotation, coupled with an instrumented shank, i.e., a scale with 70 mm of height, from the base, suitable for the measurements of the drag and sustenance by the International and English Systems. The acquisition system provides values of average speed with 0.1 m/s (0.36 km/h) and 0.05 Newtons of resolution, to the speed and drag force, respectively.

The rate of blockage in the case of small sections of tests is relevant. The correction equation used, according to Achenbach (1974):

$$C_{Dcorrected} = C_{Dinitial} \left[\frac{1}{\left(1 + 0.25 \frac{A}{C} \right)^2} \right]$$
(1)

In which A is the frontal area of the object and C is the area of test section.

Four tennis balls were used in the research, and their characteristics are shown in Tab. 1. To measure the parameters of diameter and width of the seam, it was considered the simple average number of several measurements performed on a digital pachymeter with a resolution of 0.05 mm. The mass of the ball was measured using a digital scale with a resolution of 0.001g.

Brand	Condition	Diameter (10 ⁻³ m)	Mass (10 ⁻³ kg)	Seam width (10 ⁻³ m)
Wilson 3	New	66.1(0)	57.74(2)	1.8(5)
Wilson 3	Used	63.9(1)	55.09(7)	2.0(1)
Babolat trophy	New	64.7(3)	56.70(9)	3.6(8)
Babolat trophy	Used	63.2(0)	54.41(4)	3.2(6)

Table 1. Geometric characteristics of the tennis balls

The drag coefficient, C_D , and the Reynolds number, Re, were set in accordance with Alam *et al.*(2015) and Fox *et al.*(2020):

$$C_{\rm D} = \frac{F_{\rm D}}{\frac{1}{2}\rho V^2 A}$$

$$Re = \frac{VL}{v}$$
(2)
(3)

In which F_D is the drag force on a tennis ball, ρ is the air density, 1.18 kg/m³, V is the average air speed, A is the frontal area of the object, i.e., the tennis ball, L is the characteristic length. For a tennis ball, the diameter is defined in Tab. 1, and v is the kinematic viscosity of the air at room temperature, $1.54 \cdot 10^{-5}$ m²/s.

3. RESULTS AND DISCUSSION

3.1. Position effect

Initially, the tennis balls were compared in relation to their position in the test section, i.e., the effect of the position of the seam (kerfs) on the aerodynamic properties was assessed. Each new and used ball was analyzed, in the test section, attached in the positions 1 and 2, as shown in Fig. 2, 3, 4 and 5. Position 2 is obtained by rotating position 1 in 90°. For the other possible positions, it was considered the equivalent to one of these, in other words, 180° is equivalent to the rotationed position of 0°. The 270° position is equivalent to the symmetrical of the position 2 (90°) in relation to a horizontal axis, passing through the center of the ball.

Source: Authors (2023).

Wilson 3

Figure 2. Wilson Tennis Ball 3(New), (a) position 1 e (b) position 2





Figure 4. Babolat Tennis Ball (Used), (a) position 1 and (b) position 2



Figure 5. Wilson Tennis Ball 3 (Used), (a) position 1 and (b) position 2



It is highlighted, from Fig. 4 and 5, that it is impossible to measure the time of use of each ball analyzed in the test section of the wind tunnel.

Figures 6, 7, 8, and 9 represent the drag coefficient versus the Reynolds number to the new and used ball in the positions 1 and 2. The drag coefficient and Reynolds number shown in these figures were calculated by the Eq. (3) and (2).

In these figures, the drag coefficient depends on the Reynolds number. Therefore, the effect of compressibility or the free surface effect on the drag force was neglected. Seifert (2012) analisa em seu trabalho o efeito do número de Reynolds e enfatiza a importância do estudo desse parâmetro adimensional.

Aerodynamic studies establish relations between the drag coefficient and the Reynolds number through a dimensional analysis. The Reynolds number is a dimensionless number in which the spreading speed of the fluid on the object is related to the geometry of the object.

The Figures 6 and 7 show that the drag coefficient, regardless of the position in which the ball was placed in the test section, decreases with the increase of the Reynolds number, as observed in the theory.

There is non-significant difference between positions 1 and 2. This difference, however, could not possibly affect the parameter position, for the new balls. For both balls, it is observed similarity in the curve of the drag coefficient versus the Reynolds number, i.e., there is no difference between the Wilson Ball 3 and the Babolat ball. This demonstrates the presence of frictional and pressure drags in both balls.



Figure 6. Re versus C_D to BABOLAT ball (New), Source: Authors (2023).

Figure 7. *Re* versus *C_D* to the WILSON Ball 3 (New), Source: Authors (2023).



Figures 8 and 9 are qualitatively analogous to Fig. 6 and 7, as previously drescribed. However, it is very clear that there is a difference in Fig. 9 related to the position. This figure shows higher wear and tear of this ball when compared to the ball analyzed in Fig. 8.

Figure 8 presents a drag coefficient inferior to 3.5 to $Re = 2 \cdot 10^4$, regardless the position. It is observed in Fig. 9. The numerical value of the drag coefficient is higher in Fig. 6 and 7, compared to Fig. 8 and 9, due to the presence of fuzz.

It is noteworthy that fuzz tends to annul the effects of the position of the seam. Usually, the Position 2, whose frontal area presented a larger area length of sewing, showed higher drag values for lower Reynolds number in all examined balls.



Figure 8. Re versus C_D to the BABOLAT ball (Used), Source: Authors (2023).





Figures 6, 7, 8, and 9 also highlight that, for high Reynolds numbers, greater than $3.5 \cdot 10^4$, lower drag coefficient is found, which suggests the loss of the influence of the position of the seam. This finding was observed in the studies conducted by Mehta and Pallis (2001a, 2001b) to the *Re* between 46,000 and 161,000 in two Wilson tennis balls.

The value of the drag coefficient obtained by Mehta and Pallis (2001a, 2001b) is significantly similar to those in Fig. 8 and 9, i.e., Re = 46,000.

Figures 8 and 9 shows a maximum average variation of 9% of reduction in the drag from position 1 to position 2, to the fuzzed ball. However, an average of 8% of variance was found for the drag coefficient, due to the orientation of the seam for low Reynolds numbers (below 80km/h), in the studies of Alam *et al.* (2003) and Alam *et al.* (2004a), which is an acceptable difference. These high percentages of reduction are due to the fact that the position number 1 gives a more aerodynamic shape to the ball, because of the presence of more favorable lines to the runoff, reducing the turbulent boundary layer and delaying displacement. In the balls used in this study, the difference of position

was more evident, mainly in Fig. 9. This phenomenon can be attributed to the lack of fuzz, which makes the presence of the seam more noticeable.

The high values of C_D for low *Re* leads us to discard this trend and ascribe it to an experimental error, because they become difficult to be measured, since the flow of the wind tunnel is slowed down, which requires higher sensitivity from the acquisition system. However, Mehta *et al.* (2008) compared their results to that of a smooth sphere. The total error to the drag force of the tennis balls would be lower since the drag force is higher. Thus, they carried out an analysis on a smaller scale and initially attributed these high values to the change in orientation of the filaments, which, at slow or almost null speeds, remained almost perpendicular to the surface of the ball.

They consider that, as the flow velocity increases, the filaments are forced to bend down due to aerodynamic effects of drag. Therefore, the contribution of drag is reduced due to a high Reynolds number. Besides the inclination in favor of the runoff, Mehta *et al.* (2008) re-examines each filament in relation to the Reynolds number. It is estimated that, based on the filament diameter, the estimated Reynolds number is around 20, which places it at a range in which the C_D (considering the circular wire-cylinder) is high ($C_D \approx 3$) and inversely proportional to the Reynolds number. Therefore, high values of the drag coefficient for low Reynolds numbers are assigned to the combined effect of the fuzz filament orientation and the Reynolds numbers associated with individual filaments. Therefore, it explains the achievement of the drag coefficient, C_D , ranging up to 4 for low *Re*, as shown in Fig. 6,7, 8, and 9.

It is noteworthy that several investigators achieved no results for speeds below 50 km/h, as evidenced in Fig. 6, 7, 8 and 9.

Figure 6 and 7 show a drag coefficient of around 0.60 for high values of *Re*. This fact was observed by several researchers, including Alam *et al.* (2004a), Alam *et al.* (2004b), and Alam *et al.* (2004c), who determined values of drag coefficient ranging from 0.55 to 0.65 for most new tennis balls, and Mehta and Pallis (2001a, 2001b), who found a drag coefficient of around 0.62 for new balls.

3.2. Effect of fuzz

It is not possible to carry out a direct comparison between the used and new balls for the same position with regard to fuzz, since the diameters of the balls are different and the diameter is compared with a separated study. This difference in diameter is a result of the use of these balls in competition, i.e., the balls reduce their diameter throughout their useful life due to internal pressure loss. Therefore, the comparison between balls was performed considering the same brand, position and diameter and the results are defined for balls that only differ in the presence or absence of fuzz. The methodology used in this analysis consists of rubbing the new balls, i.e., removing the fuzz, performing tests on the tunnel and comparing the results with those already obtained for the balls before the rubbing process. Figure 10 illustrates the Reynolds number versus the drag coefficient, for the analysis of the fuzz effect.

The fuzzless Wilson 3 balls analyzed in the experiment presented a slight decrease in the drag coefficient, as shown in Fig. 10. The comparison between fuzzless Wilson 3 balls and the same type of balls with fuzz, reduced drag coefficient to a value close to 10.2% is observed for low values of the Reynolds number.

In the manufacturing process, on the cover tissue of a tennis ball, the junctions of the elements of fuzz on its surface define the relative roughness on the surface of the felt, which is evident by simple observation. The elements of fuzz have finite thickness and length, thus forming, as defined Mehta *et al.* (2008), an additional porous coat in the ball, through which the air can flow. Thus, a tennis ball can be seen as a rough sphere with a porous cover. Subsequently, each element of fuzz will also experience a drag pressure. Therefore, Mehta *et al.* (2008) define "fuzz drag" as the sum of drag pressure experienced by each element of fuzz on the surface of the ball.



Figure 10. Re versus C_D for new WILSON 3 with and without fuzz, Source: Authors (2023).

3.3. Effect of diameter

A new fuzzless Wilson 3 ball and a used Wilson 3 ball (fuzz removed from usage) were used in this analysis, as shown in Fig. 11. According to Tab. 1, the analyzed balls present with a difference of approximately 3.3% in diameter. With this difference, it is possible to estimate an increase of almost 7% in the value of the drag coefficient, according to Eq. (2). Uncorrected experimental values indicated an approximate increase of 9% in the diameter. By correction calculation, Eq. (1), with an area of the test section of 32.300 mm², using the diameters from Tab. 1 and Eq. (1), the correction coefficients 0.952 and 0.949 were achieved, respectively, to the used and the new ball. Thus, the average percentage change in diameter is now almost 8% (7.86%). Evidently, if the diameter of the ball is larger, the drag force will be increased due to the higher projection of the frontal area, but a simple range of sizes should not affect C_D , since the other parameters, such as surface characteristics and fuzz have not changed significantly.



Figure 11. Re versus new and used C_D WILSON (without fuzz), Source: Authors (2023).

With roughness, the growth rate of the boundary layer is increased, resulting in early detachment and consequently higher C_D values. For high values of Re, according to Mehta *et al.* (2008), C_D is expected to reach a constant level and increase with increased roughness, in the transcritical flow, as evidenced in the measurements performed by Achenbach (1974). However, the same data from Achenbach (1974) demonstrate an upper limit for $C_D \approx 0.4$ tot spheres with increasing roughness (transcritical flow). According to Achenbach (1974), the measure for the location of the separation of this C_D value is approximately 100°. Increased superficial friction coefficient makes the boundary layer more susceptible to separation, in opposition to the tendency of the boundary layer of separating as it becomes thicker (Mehta *et al.*, 2008). Although transcritical flow is not in the scope of this research, it is noteworthy that, for certain types of roughness, a limit is reached for a C_D in this condition because the effects of increased thickness of the boundary layer are subjugated to those due to the increased superficial friction coefficient.

4. CONCLUSION

In tennis games, the ball moves with a speed ranging of between 40,000 < Re < 400,000 of Reynolds numbers. However, the main advantage observed of operating at levels below this interval or at its beginning is the observation of fuzz behavior in the early stages of the runoff, as stated in item Discussion. Thus, the flow over a tennis ball usually happens in the transcritical flow, in which the location of detachment moves significantly depending on the Reynolds number. According to Mehta *et al.* (2008), it means that the C_D is does not depend on the Reynolds number, since the total drag on a rounded body, such as a tennis ball, is almost completely caused by the pressure drag.

The high values of the drag coefficient are attributed to low Reynolds numbers, a combined effect of the orientation of the ball fuzz filaments and the effects of Reynolds numbers for each individual filament.

In the present study, the drag coefficient of the analyzed balls ranged between $C_D \approx 3$ and $C_D \approx 0.60$, for low Reynolds numbers. The average drag coefficient varies between 0.55 and 0.65 for a new tennis ball for Reynolds numbers ranging between 69,000 < Re <161,000 (60-140 km/h). However, the C_D value for a used ball is slightly lower compared to that of a new ball, for the reasons already stated. The orientation of the seam has a negligible effect on the drag coefficient at high Reynolds numbers. However, some effects were observed for low Reynolds numbers (~9% increase in the C_D value).

REFERENCES

- Achenbach, E. (1972). Experiments on the flow past spheres at very high Reynolds numbers. *Journal of fluid mechanics*, 54(3), 565-575.
- Achenbach, E. (1974). The effects of surface roughness and tunnel blockage on the flow past spheres. *Journal of fluid mechanics*, 65(1), 113-125.
- Alam, F., Chowdhury, H., & Moria, H. (2019). A review on aerodynamics and hydrodynamics in sports. *Energy Procedia*, 160, 798-805.
- Alam, F., Smith, S., Chowdhury, H., & Moria, H. (2012). Aerodynamic drag measurement of American footballs. *Procedia engineering*, 34, 98-103.
- Alam, F., Subic, A., & Watkins, S. (2003). An experimental study on the aerodynamic drag of a series of tennis balls. Sports Dynamics: Discovery and Application (edited by Subic A., Trivailo P. and Alam F.), 295-300.
- Alam, F., Subic, A., & Watkins, S. (2004a). Effects of spin on aerodynamic properties of tennis balls. *The Engineering of Sport*, 5, 83-89.
- Alam, F., Watkins, S., & Subic, A. (2004b). The effects of surface structures on aerodynamic properties of tennis balls. In *Proceedings of the 2nd BSME-ASME International Conference* on Thermal Engineering (Vol. 1, pp. 357-362).

- Alam, F., Watkins, S., & Subic, A. (2004c). The aerodynamic forces on a series of tennis balls. In *Proceedings of the 15th Australasian fluid mechanics conference* (pp. 13-17).
- Asai, T., & Kamemoto, K. (2011). Flow structure of knuckling effect in footballs. *Journal of fluids and structures*, 27(5-6), 727-733.
- Bray, K., & Kerwin, D. (2003). Modelling the flight of a soccer ball in a direct free kick. *Journal of sports sciences*, 21(2), 75-85.
- Choppin, S. (2013). Calculating football drag profiles from simulated trajectories. *Sports Engineering*, *16*(3), 189-194.
- Djamovski, V., Pateras, J., Chowdhury, H., Alam, F., & Steiner, T. (2012). Effects of seam and surface texture on tennis balls aerodynamics. *Procedia Engineering*, *34*, 140-145.
- Driscoll, H., Hart, J., & Allen, T. (2016). Use of image based sports case studies for teaching mechanics. *Procedia engineering*, 147, 884-889.
- Fox, R. W., McDonald, A. T., & Mitchell, J. W. (2020). Fox and McDonald's introduction to fluid mechanics. John Wiley & Sons. FOX
- Goff, J. E., & Carré, M. J. (2009). Trajectory analysis of a soccer ball. American Journal of *Physics*, 77(11), 1020-1027.
- Haake, S. J., Chadwick, S. G., Dignall, R. J., Goodwill, S., & Rose, P. (2000). Engineering tennis– slowing the game down. *Sports Engineering*, *3*(2), 131-143.
- Kozlov, A., Chowdhury, H., Mustary, I., Loganathan, B., & Alam, F. (2015). Bio-inspired design: aerodynamics of boxfish. *Procedia engineering*, *105*, 323-328.
- Mehta, R. D., & Pallis, J. M. (2001a). Sports ball aerodynamics: effects of velocity, spin and surface roughness. *Minerals, Metals and Materials Society/AIME, Materials and Science in Sports* (USA),, 185-197.
- Mehta, R. D., & Pallis, J. M. (2001b). The aerodynamics of a tennis ball. *Sports Engineering*, 4(4), 177-189.
- Mehta, R., Alam, F., & Subic, A. (2008). Review of tennis ball aerodynamics. Sports Technology. *John Wiley and Sons Asia Pte Ltd*, *1*, 1-16.
- Moria, H., Chowdhury, H., Alam, F., & Subic, A. (2011). Aerodynamic behaviour of stretchable sports fabrics. *Sports Technology*, 4(3-4), 171-177.
- Seifsert, J. (2012). A review of the Magnus effect in aeronautics. *Progress in Aerospace Sciences*, 55, 17-45.

RESPONSIBILITY NOTICE

The author(s) is (are) the only responsible for the printed material included in this paper.