The Journal of Engineering and Exact Sciences – jCEC, Vol. 09 N. 04 (2023)
journal homepage: https://periodicos.ufv.br/jcec
eISSN: 2527-1075
ISSN: 2446-9416

Transient Thermal Cooling of Electronics Systems using Functional Graded Fins: Hybrid Computational Analysis

Article Info:
Article history: Received 2023-01-25 / Accepted 2023-05-07 / Available online 2023-05-24
doi: 10.18540/jcecvl9iss4pp15810-01e

Akinwale Emmanuel Jemiseye
ORCID: https://orcid.org/0009-0009-1772-0606
Department of Computer Engineering, University of Louisiana, Lafayette, USA
E-mail: akinwale.jemiseye1@louisiana.edu
Gbeminiyi Musibau Sobamowo
ORCID: https://orcid.org/0000-0003-2402-1423
Department of Mechanical Engineering, University of Lagos
Lagos, Nigeria
E-mail: gsobamowo@unilag.edu.ng
Olayiwola Olatunji Mayowa
ORCID: https://orcid.org/0000-0002-4285-080X
Department of Petroleum Engineering, University of Louisiana, Lafayette, USA
E-mail: olatunji.olayiwola1@louisiana.edu

Abstract
Passive cooling of electronics systems requires the use of fins and spines. Improved heat transfer enhancements in the systems have been achieved through porosity, magnetic field, functionally graded materials, etc. However, the space and time-dependent analysis of passive device with inherent nonlinearities and explorations of conditions at the tip have not been fully presented. Therefore, this work presents an application of hybrid computation analysis using combined methods of Laplace transformation and Legendre wavelet collocation to the space and time-dependent nonlinear thermal models of a longitudinal conductive-radiative functionally graded extended surface. The spatial-dependent thermal conductivity is considered for three different cases. The present analysis results demonstrated good agreements with numerical results. The significances of the model pertinent parameters on the fin's performance are thoroughly considered and scrutinized by the aid of graphical illustrations. The fin tip thermal response decreases with increase in conductive-convective parameter, but it increases as time progresses. The in-homogeneity index is directly proportional to the FGM fin temperature. However, the convective term is inversely proportional to the fin's thermal distribution. The fin temperature under quadratic-law heat conductivity of the passive device shows an enhanced performance of lower thermal distribution than the linear- and exponential-law heat conductivity. Ultimately, the study provides a very useful in the design of the fin with FGM.
Keywords: Electronic cooling; Thermal-management; Convective-radiative fin; Functionally graded materials (FGMs); Fin tip temperature; Laplace Transform-Legendre Wavelet Collocation Method (.LT-LWCM).

1. Introduction

The increase in the integrated and additional functionalities of electronic systems leads to increase heat flux in the systems. Also, the miniaturisation of electronic devices and equipment inadvertently results and increases in thermal stress of electronic systems. Such thermal stress causes fail prematurely of the system if not properly managed. Therefore, there are needs for effective cooling of their embedded electronic circuits (Fig. 1) for their efficient performance. The cooling of the electronics prevents short-circuiting which results in Ohmic heating. Such type of heating damages the electronic devices, causes voids and explosions in the system. However, through effective thermal dissipation and management, the thermal damages of the systems can be obviated, and the performances of the systems are optimised. Additionally, the costs of preventive maintenance of the systems are saved.

Fig. 1 Electronic board

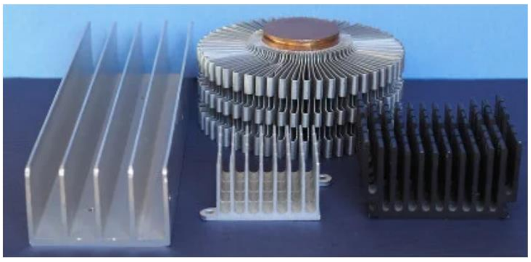


Fig.2 The passive devices for the electronic cooling

Different methods of electronic cooling have been put forward. Choosing the most effective cooling technique comes with the consideration of multiple factors such as the systems maintenance, design flexibility, noise, vibration, and reliability. Thermal enhancements and augmentations in thermal and electronics have been passively achieved by fins and spines (Fig. 2). The thermal characterizations of the passive devices have been well explored (Kiwan & Nimr, 2001; Kiwan, 2007a & 2007b; Kiwan & Zeitoun, 2008; Abbasbandy *et al*., 2011; Bhanja & Kundu, 2011; Gorla and Bakier, 2011; Shivanian & Hashim, 2011; Saedodin & Olank, 2011; Petroudi *et al*. 2012; Kundu *et al*., 2012; Moradi *et al*., 2014; Saedodin & Shahbabaei, 2013; Hatami & Ganji 2013; Hatami et al., 2013; Ghasemi *et al*., 2014; Hatami *et al*., 2014; Moradi *et al*., 2014; Rostamiyan *et al*., 2014; Darvishi *et al*. 2015; Sobamowo, 2016).

The studies are based on homogeneous passive devices for electronics. Such components have materials which their thermal conductivities are assumed constants or temperature dependent. However, there are cases where the thermal conductivity of the fins is coordinate- or spatial dependent such as fins with functionally graded materials (FGM). The behaviours and performance of these fins have been studied (Noda, 1999; Jin, 2002; Sladek *et al*., 2003; Chen & Tong, 2004; Aziz, 2005; Eslami *et al*., 2005; Hosseini *et al.*, 2007; Babaei & Chen, 2008; Aziz & Rahman, 2009; Khan & Aziz, 2012; Gaba *et al.*, 2016; Hassanzadeh & Pekel, 2016; Oguntala *et al.*, 2019; Sobamowo *et al.*, 2019a; Sobamowo *et al.*, 2019b; Oguntala, *et al.*, 2022; Sahu & Bhowmick, 2022).

The thermal characterizations have attracted various mathematical methods for the mathematical analysis. However, the solutions of these methods are inconvenient especially for practical applications (Sobamowo, 2016). Additionally, such unpalatable situation is deepened when the methods are utilized for solving nonlinear transient or PDE problems bring about further complications in the development of an easily implementable solutions. Therefore, there is a need for alternative approach that comes with simplicity, flexibility, generalization, preciseness and robustness coupled with low computational cost and time. Therefore, in the present letter, Legendre wavelet collocation method (LWCM) is sought because of its all-inclusiveness of the aforementioned mathematical advantages. For the transient heat flow problem, Laplace transform

(LT)-LWCM to analyze the functionally graded fins. Also, for the improvement in the cooling capacity of the device, the fin is made of functionally graded materials (FGMs). Sensitivity analyses are conducted and presented through MATLAB graphical display.

2. Model Presentation

Fig. 1 shows a solid longitudinal fin with FGM that is subjected to thermal convective and radiation effects as shown (Fig.3.). Except for the spatial-dependent thermal conductivity, the properties of the fin material are assumed to be directionally, thermally and spatially invariant and the heat flows unidirectionally in the device. Therefore, the uni-directional thermal energy equation is given as

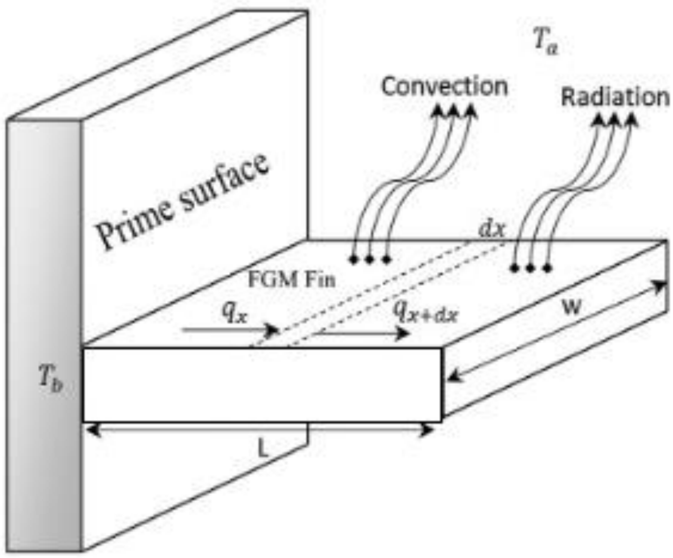


Fig. 3 Schematic diagram of the radiative-convective rectangular FGM fin

$$\frac{\partial}{\partial \breve{x}}\left(k\left(\breve{x}\right)\frac{\partial \breve{T}}{\partial \breve{x}}\right)-hA^{-1}P(\breve{T}-\breve{T}_a)-PA^{-1}\varepsilon\sigma(\breve{T}^4-\breve{T}_a^4) \;=\; c_p\rho\frac{\partial \breve{T}}{\partial \breve{t}} \tag{1}$$

The prevailing conditions initially and at the boundaries are

$$\breve{T}(\breve{x},0)=\breve{T}_o, \tag{2}$$

$$\frac{\partial \breve{T}(0,\breve{t})}{\partial \breve{x}}=0\,, \tag{3}$$

$$\breve{T}(L,\breve{t})=\breve{T}_b, \tag{4}$$

Three cases of spatial-dependent heat conductivity (linear-, quadratic- and exponential-laws) of the FG fin are presented:

Case 1

$$k(\breve{x})=k_o\left(1+\gamma_l\breve{x}\right) \tag{5}$$

Case 2

$$k(\breve{x}) = k_o\left(1+\gamma_q \breve{x}^2\right) \tag{6}$$

Case 3

$$k(\breve{x}) = k_o e^{\gamma_i \breve{x}} \tag{7}$$

When Eqs. (5), (6) and (7) are put into Eq. (1), then for

Case 1

$$\frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + \gamma_l \breve{x} \frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + \gamma_l \frac{\partial \breve{T}}{\partial \breve{x}} - hk_o^{-1}A^{-1}P(\breve{T}-\breve{T}_a) - P\varepsilon\sigma k_o^{-1}A^{-1}(\breve{T}^4-\breve{T}_a^4) = \rho k_o^{-1}c_p \frac{\partial \breve{T}}{\partial \breve{t}} \tag{8}$$

Case 2

$$\frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + \gamma_l \breve{x}^2 \frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + 2\gamma_l \breve{x} \frac{\partial \breve{T}}{\partial \breve{x}} - hk_o^{-1}A^{-1}P(\breve{T}-\breve{T}_a) - P\varepsilon\sigma k_o^{-1}A^{-1}(\breve{T}^4-\breve{T}_a^4) = \rho k_o^{-1}c_p \frac{\partial \breve{T}}{\partial \breve{t}} \tag{9}$$

Case 3

$$\frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + e^{\gamma_i \breve{x}} \frac{\partial^2 \breve{T}}{\partial \breve{x}^2} + \gamma_i e^{\gamma_i \breve{x}} \frac{\partial \breve{T}}{\partial \breve{x}} - hk_o^{-1}A^{-1}P(\breve{T}-\breve{T}_a) - P\varepsilon\sigma k_o^{-1}A^{-1}(\breve{T}^4-\breve{T}_a^4) = \rho k_o^{-1}c_p \frac{\partial \breve{T}}{\partial \breve{t}} \tag{10}$$

The adimensional variables are given in Eq. (11),

$$\begin{gathered} X = L^{-1}x,\ \ \theta = \left(T_b - T_a\right)^{-1}\left(T^{**} - T_a\right),\ \ \tau = k_o \rho^{-1} c_p^{-1} L^{-2} t,\ \ \beta = L\gamma_l = L\gamma_i,\ \ \lambda = L^2 \beta_q, \\ Nc^2 = PA^{-1}k_o^{-1}L^2 h,\ \ Nr = 4A^{-1}k_o^{-1}L^2 \varepsilon\sigma T_a^3 P,\ \ \Omega = \left(T_b - T_a\right)^{-1} T_a \end{gathered} \tag{11}$$

where
A, h, cp, k, ρ, L, Nc, Nr, P, t, T, Tb, Ta, x, X and θ are the fin area, coefficient of heat transfer, specific heat/thermal capacity, thermal/heat conductivity, fin material density, fin length, unitless convective parameter, unitless radiative parameter, fin perimeter, fin thickness, fin temperature, fin base temperature, ambient/sorrounding temperature, fin longitudinal length and fin unitless length and fin unitless temperature. Also**,** β is defined as the in-homogeity index of the fin.

The adimensional forms of the above equations (8)-(10) are

Case 1

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2} + \beta \frac{\partial \theta}{\partial X} + \beta X \frac{\partial^2 \theta}{\partial X^2} - Nc^2\theta - 4Nr\Omega^3\theta - 6Nr\Omega^2\theta^2 - 4Nr\Omega\theta^3 - Nr\theta^4 \tag{12}$$

Case 2

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial X^2}+X^2\lambda\frac{\partial^2\theta}{\partial X^2}+2X\lambda\frac{\partial\theta}{\partial X}-Nc^2\theta-4Nr\Omega^3\theta-6Nr\Omega^2\theta^2-4Nr\Omega\theta^3-Nr\theta^4 \tag{13}$$

Case 3

$$\frac{\partial\theta}{\partial\tau}=e^{\beta X}\frac{\partial^2\theta}{\partial X^2}+\beta e^{\beta X}\frac{\partial\theta}{\partial X}-Nc^2\theta-4Nr\Omega^3\theta-6Nr\Omega^2\theta^2-4Nr\Omega\theta^3-Nr\theta^4 \tag{14}$$

The adimensional conditions at the initial stage and boundaries are

$$\theta(X,0)=0, \tag{15}$$

$$\frac{\partial\theta(0,\tau)}{\partial X}=0, \tag{16}$$

$$\theta(1,\tau)=1, \tag{17}$$

3. Solution Methodology

The solution methodology involves applications of Laplace transform-Legendre wavelet collocation method (LT-LWCM). The procedures are presented as follows:

Application of LT to Eqs. (12), (13) and (14) provides the following solutions

Case 1

$$\frac{d^2\tilde{\theta}}{dX^2}+\beta\frac{d\tilde{\theta}}{dX}+\beta X\frac{d^2\tilde{\theta}}{dX^2}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \tag{18}$$

Collecting like terms, we have

$$\frac{d^2\tilde{\theta}}{dX^2}+\beta\frac{d\tilde{\theta}}{dX}+\beta X\frac{d^2\tilde{\theta}}{dX^2}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \tag{19}$$

Case 2

$$\frac{d^2\tilde{\theta}}{dX^2}+2X\lambda\frac{d\tilde{\theta}}{dX}+\lambda X^2\frac{d^2\tilde{\theta}}{dX^2}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \tag{20}$$

Bringing like terms together, we have

$$\frac{d^2\tilde{\theta}}{dX^2}+X^2\lambda\frac{d^2\tilde{\theta}}{dX^2}+2X\lambda\frac{d\tilde{\theta}}{dX}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \quad (21)$$

Case 3

$$e^{\beta X}\frac{d^2\tilde{\theta}}{dX^2}+\beta e^{\beta X}\frac{d\tilde{\theta}}{dX}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \quad (22)$$

When the like terms are collected, we have

$$e^{\beta X}\frac{d^2\tilde{\theta}}{dX^2}+\beta e^{\beta X}\frac{d\tilde{\theta}}{dX}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \quad (23)$$

In the Laplace domain, the adimensional conditions at the initial stage and boundaries are

$$\tilde{\theta}(X,0)=0, \quad (24)$$

$$\frac{\partial\tilde{\theta}(0,s)}{\partial X}=0, \quad (25)$$

$$\theta(1,s)=\frac{1}{s}, \quad (26)$$

3. The Solution Methodology: Analytical-Numerical methods using LT-LWCM
Semi-numerical solution is adopted using LT-LWCM.

3.1 Analytical solutions of the temporal part of the model

Application of LT to Eqs. (12), (13) and (14) provides the following solutions

Case 1

$$\frac{d^2\tilde{\theta}}{dX^2}+\beta\frac{d\tilde{\theta}}{dX}+\beta X\frac{d^2\tilde{\theta}}{dX^2}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \quad (18)$$

Collecting like terms, we have

$$\frac{d^2\tilde{\theta}}{dX^2}+\beta\frac{d\tilde{\theta}}{dX}+\beta X\frac{d^2\tilde{\theta}}{dX^2}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \quad (19)$$

Case 2

$$\frac{d^2\tilde{\theta}}{dX^2}+X^2\lambda\frac{d^2\tilde{\theta}}{dX^2}+2X\lambda\frac{d\tilde{\theta}}{dX}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \quad (20)$$

Bringing like terms together, we have

$$\frac{d^2\tilde{\theta}}{dX^2}+X^2\lambda\frac{d^2\tilde{\theta}}{dX^2}+2X\lambda\frac{d\tilde{\theta}}{dX}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \tag{21}$$

Case 3

$$e^{\beta X}\,\frac{d^2\tilde{\theta}}{dX^2}+\beta e^{\beta X}\,\frac{d\tilde{\theta}}{dX}-Nc^2\tilde{\theta}-4Nr\Omega^3\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=s\tilde{\theta} \tag{22}$$

When the like terms are collected, we have

$$e^{\beta X}\,\frac{d^2\tilde{\theta}}{dX^2}+\beta e^{\beta X}\,\frac{d\tilde{\theta}}{dX}-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\tilde{\theta}-6Nr\Omega^2\tilde{\theta}^2-4Nr\Omega\tilde{\theta}^3-Nr\tilde{\theta}^4=0 \tag{23}$$

In the Laplace domain, the adimensional conditions at the initial stage and boundaries are

$$\tilde{\theta}(X,0)=0, \tag{24}$$

$$\frac{\partial\tilde{\theta}(0,s)}{\partial X}=0, \tag{25}$$

$$\theta(1,s)=\frac{1}{s}, \tag{26}$$

3.2 Numerical Solutions of the Nonlinear Model using LWCM

Eqs. (19), (21) and (23) are nonlinear equations, in order to obtain solution to the nonlinear equation, LWCM. The principle and the procedure are described in our previous work (Sobamowo, 20170). Following the LWCM procedures as presented by Sobamowo (2017), we have the following numerical solutions

Case 1

$$\begin{aligned}&C^T\tilde{\psi}(X)+\beta\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)+C^T\tilde{P}^2\tilde{\psi}(X)\right)+\beta XC^T\tilde{\psi}(X)-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)\\&-6Nr\Omega^2\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^2-4Nr\Omega\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^3\\&-Nr\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^4=R\left(X,c_1,c_2,.....,c_n\right)\end{aligned} \tag{27}$$

Case 2

$$C^T\tilde{\psi}(X)+\lambda X^2C^T\tilde{\psi}(X)+2\lambda X\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)+C^T\tilde{P}^2\tilde{\psi}(X)\right)-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)$$
$$-6Nr\Omega^2\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^2-4Nr\Omega\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^3$$
$$-Nr\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^4=R(X,c_1,c_2,.....,c_n) \tag{28}$$

Case 3

$$e^{\beta X}C^T\tilde{\psi}(X)+\beta e^{\beta X}\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)+C^T\tilde{P}^2\tilde{\psi}(X)\right)-\left(s+Nc^2\tilde{\theta}+4Nr\Omega^3\right)\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)$$
$$-6Nr\Omega^2\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^2-4Nr\Omega\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^3$$
$$-Nr\left(\frac{1}{s}-C^T\tilde{P}^2\tilde{\psi}(1)d'\tilde{P}\tilde{\psi}(X)+C^T\tilde{P}^2\tilde{\psi}(X)\right)^4=R(X,c_1,c_2,.....,c_n) \tag{29}$$

Where

$$C=\begin{bmatrix} c_1,1,......,c_1,M-1 \\ c_2,1,......,c_2,M-1 \\ . \\ . \\ . \\ c_{2^{k-1}},1,...,c_{2^{k-1}},M-1 \end{bmatrix}$$

$$\tilde{\psi}(X)=\begin{bmatrix} \tilde{\psi}_{1,0}(X),\tilde{\psi}_{1,1}(X),...,\tilde{\psi}_{1,M-1}(X) \\ \tilde{\psi}_{2,0}(X),\tilde{\psi}_{2,1}(X),......,\tilde{\psi}_{2,M-1}(X) \\ . \\ . \\ . \\ \tilde{\psi}_{2^{k-1},0}(X),\tilde{\psi}_{2^{k-1},1}(X),....,\psi_{2^{k-1},M-1}(X) \end{bmatrix} \tag{30}$$

$$P=\frac{1}{2}\begin{pmatrix} 1 & \frac{1}{3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & & & \vdots \\ \vdots & \vdots & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \ddots & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \frac{-\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \end{pmatrix}$$

$R(X, c_1, c_2, ..., c_n)$ is the residue.

The resulting simultaneous algebraic equations are solved after finding the LT numerically. Also, the corresponding values of C for the different cases and for different combinations of model parameters are found through the terminal boundary condition in Eq. (17). Hence, the solutions of adimensional thermal response in the fin are obtained.

5. Result and Discussion

The resulting system of algebraic equations are simulated with MATLAB and parametric studies were explored as shown in Fig. 4-10. Also, the results of the semi-numerical solution are verified numerically using finite difference method (FDM) as illustrated (Table 1).

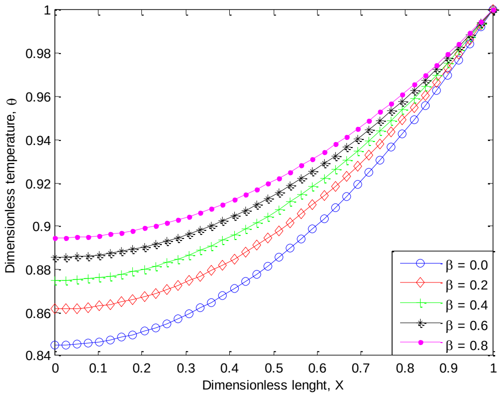
Table 1: Results of FDM and LT-LWCM

X	FDM	LT-LWCM
0.00	0.459185	0.461000
0.20	0.477670	0.478400
0.40	0.534612	0.536200
0.60	0.634595	0.635400
0.80	0.785670	0.786500
1.00	1.000000	1.000000

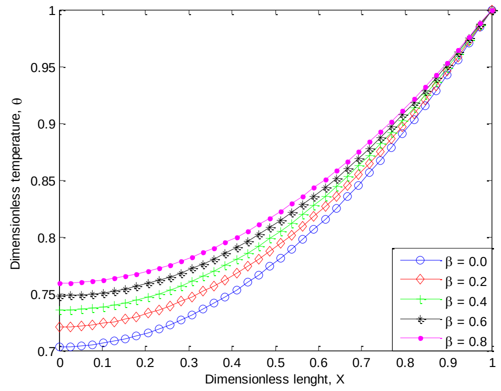
Table 2: Effects of β *and* N

N	β	λ (Tip temperature)
1	0.0	0.5784
1	0.4	0.7238
1	1.0	0.8746
2	0.0	0.3842
2	0.4	0.4567
2	1.0	0.5879
3	0.0	0.1729
3	0.4	0.2318
3	1.0	0.3492
4	0.0	0.0924
4	0.4	0.1145
4	1.0	0.1356

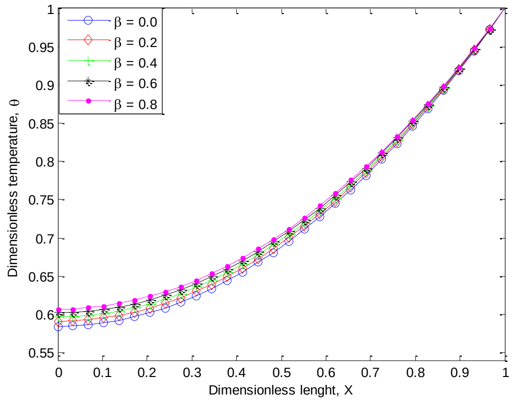
Table 2 illustrates the impacts of N and β on fin tip response. There is direct variation of β with λ. However, inverse variation N with λ. The significance of β on FG fin temperature is demonstrated in Fig. 4. The in-homogeneity index, β is directly proportional to the FGM fin temperature as depicted Fig 4a and 4b. However, the convective term is inversely proportional to the fin distribution as illustrated Fig. 4a-d but the fin temperature change becomes insignificant for large values of N (Fig. 4d). This same trend is recorded for the radiative term. For effective thermal enhancement in the heat sink/passive device, it is recommended that passive devices should be made of FGM.



(a)



(b)



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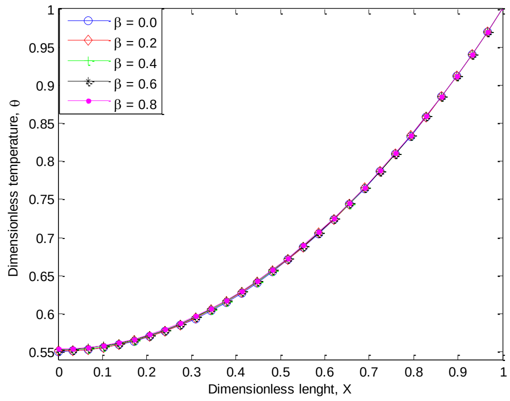


(d)

Fig. 4. Significance of β on θ when (a) Nc=1 (b) Nc =2 (c) Nc= 5 (4) Nc=15

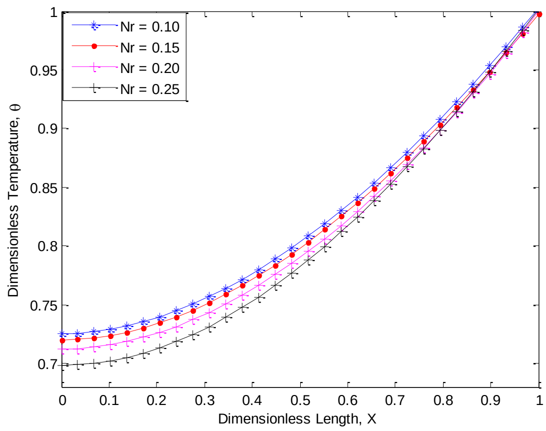


Fig. 5 Significance of Nr on θ

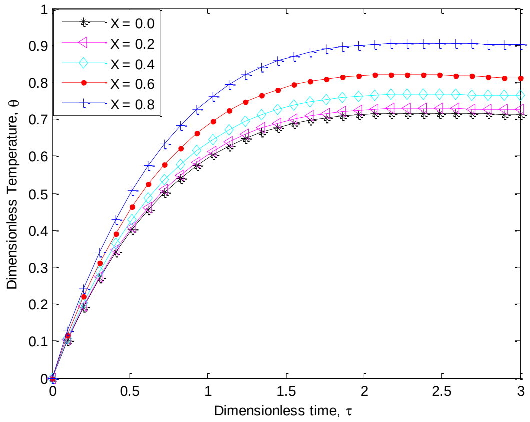


Fig. 6. Time evolution of θ at different values of *X* for linear--law spatial-dependent thermal conductivity

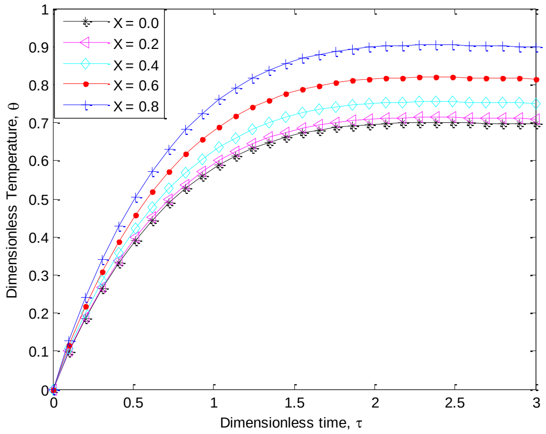


Fig. 7. Time evolution of θ at different values of *X* for quadratic--law spatial-dependent thermal conductivity

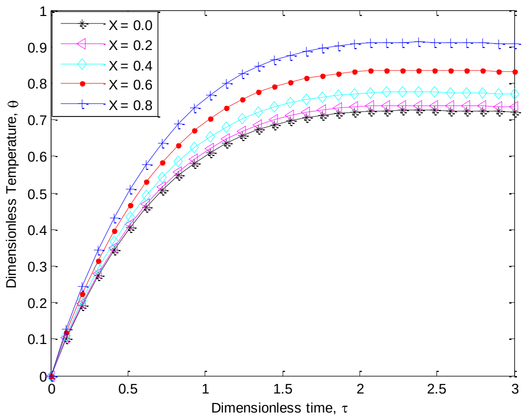
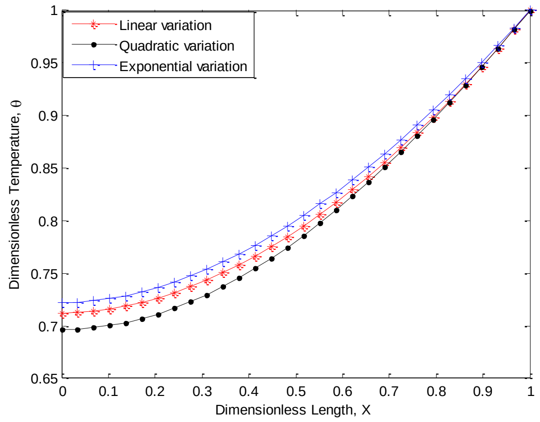


Fig. 8. Time evolution of θ at different values of *X* for exponential--law spatial-dependent thermal conductivity



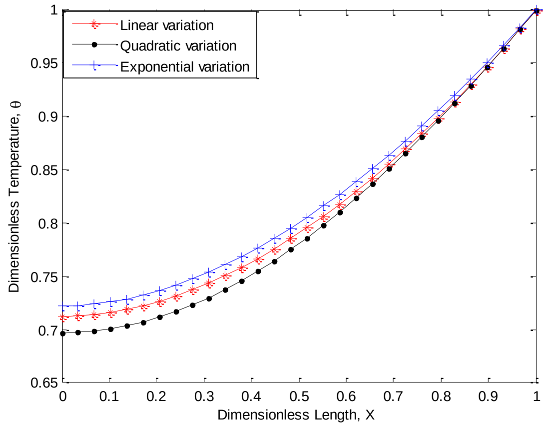
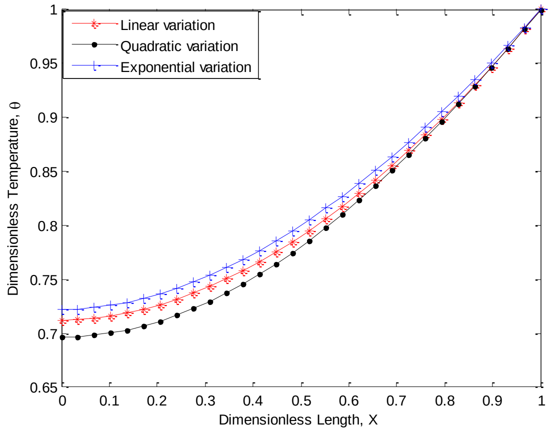


Fig. 9a Variation of θ when τ = 0.1 Fig. 9b Variation of θ when τ = 0.15



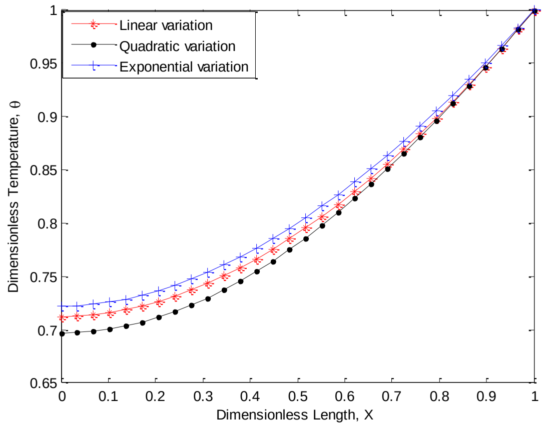


Fig. 9c Variation of θ when τ = 0.30 Fig. 9d Variation of θ when τ = 0.35

Fig. 5 presents the importance of radiation parameter (*Nr*) on fin thermal response. As in the case of the convective term, the radiative term is inversely proportional to the fin temperature. The recommendation is that the thermal performance can be enhanced by the thermal radiation in the heat augmentation. The thermal histories at different positions under different laws of the FGM fin at different positions are given in Figs. 6, 7 and 8. There is an increase in thermal distribution as time progresses but converges to a steady state for longer time.

Figs. 9a-d illustrate the impact of β on thermal characteristics of the device. The device temperature under quadratic-law variation of the passive device shows an enhanced performance of lower thermal/heat distribution than the linear- and exponential-law variation.

6. Conclusion

Laplace transformation-Legendre wavelet collocation methods have been used in this letter to scrutinize the thermal responses of functionally graded fin. The thermal/heat conductivity of the FGM have been considered for three different cases. The results of the hybrid method were verified numerically. Also, importance of the device pertinent parameters on its responses were thoroughly considered and scrutinized by the aid of graphical illustrations. The sensitivity analysis depicted that

i. the fin tip thermal response decreases with increase in conductive-convective parameter, but it increases as time progresses.
ii. The in-homogeneity index is directly proportional to the FGM fin temperature.
iii. The convective term is inversely proportional to the fin's thermal distribution.

iv. The fin temperature under quadratic-law heat conductivity of the passive device shows an enhanced performance of lower thermal distribution than the linear- and exponential-law heat conductivity.

The present study will be greatly utilized for the the design of the fin in the thermal management of electronics.

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