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Firefly Algorithm Optimization-Based LQR Controller for 1/4 Vehicle Active Suspension System: Design and Performance Evaluation

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Resumo
Os resultados desta pesquisa contribuem significativamente para o corpo existente de conhecimento sobre técnicas de otimização meta-heurística para sistemas de suspensão ativa. Ao realizar investigações e análises minuciosas, o estudo demonstra de forma eficaz as notáveis vantagens do algoritmo Firefly na otimização do desempenho de controladores convencionais e inteligentes para sistemas de suspensão ativa. Essas descobertas destacam o potencial do algoritmo para revolucionar o campo e abrir caminho para estratégias de controle mais eficientes e robustas.
A eficácia demonstrada do algoritmo Firefly em minimizar o erro entre os deslocamentos do carro e as perturbações da estrada é especialmente notável. Essa conquista garante um rastreamento mais preciso e acurado da trajetória desejada, independentemente do sinal de entrada utilizado, seja um impulso, onda senoidal ou sinal graduado em etapas.
Palavras-chave: Suspensão ativa. LQR. Algoritmo Firefly. Otimização.

Abstract
The outcomes of this research significantly contribute to the existing body of knowledge on metaheuristic optimization techniques for active suspension systems. By conducting thorough investigations and analyses, the study effectively demonstrates the remarkable advantages of the Firefly algorithm in optimizing the performance of both conventional and intelligent controllers for active suspension systems. These findings highlight the algorithm's potential to revolutionize the field and pave the way for more efficient and robust control strategies. The demonstrated effectiveness of the Firefly algorithm in minimizing the error between the car's displacements and the disturbances of the road is particularly noteworthy. This achievement ensures a more precise and accurate tracking of the desired trajectory, regardless of the input signal used, whether it be an impulse, sine wave, or step-wise graded signal.
Keywords: Active suspension. LQR. Firefly Algorithm. Optimization.

1. Introduction

The primary objective for the design of vehicle suspensions is to ensure passenger comfort and safety (Louam, 1996), as well as vehicle levitation and isolation from induced vibrations during travel (HROVAT, 1997) (Maurya & Bhangal, 2018). Depending on the vehicle and its intended use, suspensions are generally categorized as passive, semi-active, or fully active, differing in the fixed or variable values of the suspension system components (springs and/or dampers) (Senthil Kumar & Vijayrangan, 2006) (Aizuddin Fahmi *et al*., 2018).

Recently, with the advancement of digital technology driven by microprocessor-based devices, vehicle suspensions are controlled by highly sophisticated electronic regulators. These regulators serve to control and/or adjust the actuation force to achieve the objectives of their design (Savaresi *et al*., 2010). This type of suspension is known as active or semi-active suspension, consisting of hydraulic (pneumatic) or electromagnetic dampers (Aizuddin Fahmi *et al*., 2018). These suspensions are typically equipped with traditional regulators (PID, LQR, SMC, etc.) (Peng *et al*., 1997) (Conde *et al*., 2011) (Kaleemullah *et al*., 2012) (Ab Talib *et al*., 2015) (Rao & Kumar, 2015) (Sun *et al*., 2020) (Nguyen & Nguyen, 2022), intelligent regulators (Fuzzy Logic, Neural Networks, etc.) (Hasbullah & Faris, 2010) (Nagarkar *et al*., 2019), and/or hybrid and metaheuristic regulators (Mahmoodabadi *et al*., 2018) (Lavasani & Doroudi, 2020) (Wu *et al*., 2021).

Metaheuristic optimization methods have recently gained significant attention from the international research community due to their effectiveness in various optimization problems. Several widely used methods are mentioned in Xin-She Yang's book (Yang X., 2008): GWO (Grey Wolf Optimizer), FA (Firefly Algorithm), PSO (Particle Swarm Optimization), ABA (Ant & Bee Algorithms), DE (Differential Evolution), and others (Yang X.-S., 2014) (Okwu & Tartibu, 2021). However, among these problems, one can find the optimization and identification of parameters for both conventional and intelligent regulators applied to different systems. These include various applications such as machine learning, engineering applications (controller design, trajectory planning in robotics and scheduling, environmental modeling, bioinformatics and medical applications, image processing, applications in energy systems, etc.). For example, optimization of the grey wolf optimizer (GWO) for various applications (Faris *et al*., 2018) (Hatta *et al*., 2019) (Al-Khazraji, 2022), optimization of PID controller using the firefly algorithm for speed control of a DC motor (Pal *et al*., 2015) and a spinning machine (Jadhav Vilas & Asutkar, 2021), optimization of LQR controller for a wheeled mobile robot (ABUT & HÜSEYİNOĞLU, 2019) and for microalgae cultivation (Setyowati & Mardlijah, 2020), improved firefly algorithm applied to a dual-tank system compared with PSO (Selamat *et al*., 2019), modified chaos theory-based firefly algorithm for parameter identification of fractional order PID controller applied to a CSTR system (Ravari & Yaghoobi, 2019), and modification in the hierarchy of the algorithm for parameter identification of an asynchronous machine (Dif *et al*., 2020). PSO is one of the most commonly used metaheuristic methods in various applications since its invention in the mid-1990s (Gad, 2022), such as LQR and PID controller parameterization compared with double-tank control (Selamat *et al*., 2015), stabilization of a two-wheel chair using LQR controller (Aula *et al*., 2015), and optimal position control of a permanent magnet DC motor compared with the famous ACO (Ant Colony Optimizer) metaheuristic method (Rasheed, 2020). Another notable method is DE, used for PID-Fuzzy controller optimization applied to a semi-active suspension system with Magneto-Rheological damper (AHMED, *et al*., 2022), as well as new methods like the Aquila optimizer used for shifting nonlinear phenomena in boost converters (Korich, *et al*., 2023).

The context of this article is the design of an optimal LQR controller for a 1/4 active vehicle suspension using the Firefly Algorithm (FA) as a metaheuristic method for optimizing the controller's weighting matrices. This work aims to enhance the behaviour of active suspensions, improving passenger comfort and safety. In the following sections, we first define the mathematical model of the vehicle suspension, while comparing different types of suspensions (passive and active). We then provide a brief overview of the basic theories of LQR regulators and the Firefly algorithm. Subsequently, we simulate passive and active suspension models, comparing the results of LQR and LQR-FA controllers in terms of predefined comfort-safety objectives for passengers by applying different road signal patterns.

2. Modeling of quarter vehicle suspension

The objective of modeling such a physical system is to develop mathematical equations or graphical schemes that allow for understanding their behavior and facilitate their analysis, control, and/or diagnosis. To achieve this, various types of models exist, depending on the system's complexity and its input/output signals. These models include differential equations, transfer functions, state-space systems, flow graphs, bond graphs, and more.

Our vehicle suspension system has garnered significant attention in terms of research on its modeling, control/command, both theoretically and practically. Three types of vehicle suspensions can be distinguished: passive, semi-active, and active (Aizuddin Fahmi *et al*., 2018).

2.1. Passive Suspension:

In general, vehicle suspension modeling involves representing the suspension as a mass, which represents 1/4 of the vehicle, including the passenger, suspended by a passive spring and damper (with fixed values), as shown in Figure 1(a).

2.2. Semi-Active Suspension:

The semi-active vehicle suspension is a suspension system equipped with an adjustable damper, including the spring, as illustrated in Figure 1(b). This type of suspension enables dynamic adjustments to the damping characteristics, allowing it to adapt to real-time inputs and road conditions. By incorporating an adjustable damper, the semi-active suspension system offers improved comfort and stability compared to passive suspensions, as it can optimize its performance according to varying driving conditions and vehicle dynamics.

2.3. Active Suspension

In contrast, the active suspension involves the addition of an active actuator, which provides the active force required to reduce disturbances induced by the road, as shown in Figure 1(c). This type of suspension goes beyond the capabilities of semi-active and passive suspensions by actively controlling and adjusting the forces acting on the vehicle. The active actuator enables the suspension system to respond in real-time to road conditions and external inputs, thereby enhancing passenger comfort and vehicle stability. By actively counteracting disturbances, the active suspension system offers superior performance and adaptability compared to passive and semi-active suspensions.

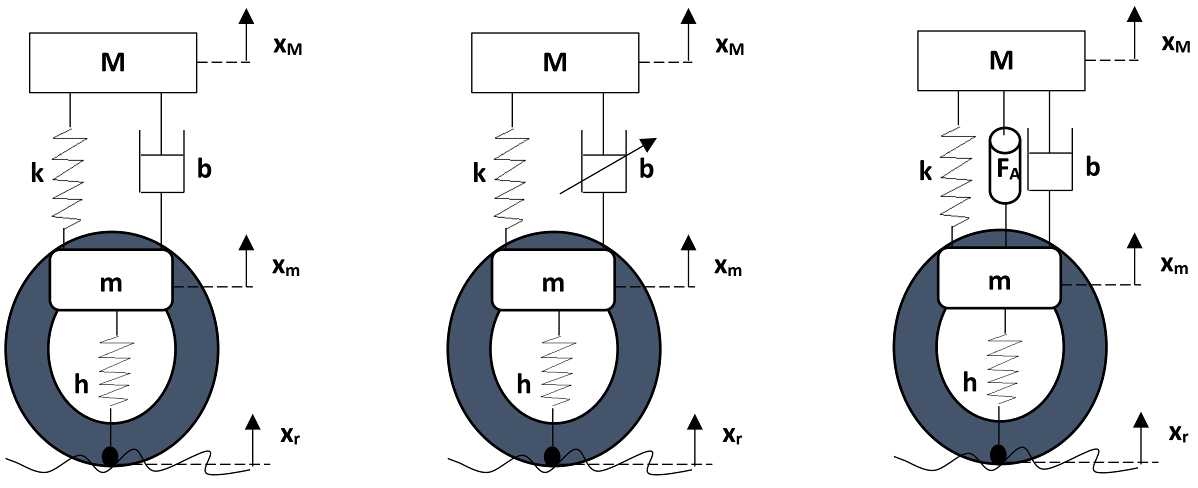

Figure 1 – Quarter Vehicle Suspension (a) Passive (b) Semi-Active (c) Active

2.4. Mathematical model

According to the laws of mechanics (Newton's second law), the following relationships are described (Ogata, 2010):

$$M\ddot{x}_M + k(x_M - x_m) + b(\dot{x}_M - \dot{x}_m) - F_A = 0 \quad (1)$$

$$m\ddot{x}_m + k(x_m - x_M) + b(\dot{x}_m - \dot{x}_M) + h(x_m - x_r) + F_A = 0 \quad (2)$$

After a change of variables, the model in state-space form is given as follows:

$$\begin{cases} \underbrace{\begin{pmatrix} \dot{x}_1 = \dot{x}_M \\ \dot{x}_2 = \dot{x}_m \\ \dot{x}_3 = \ddot{x}_M \\ \dot{x}_4 = \ddot{x}_m \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M} & \frac{k}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k}{m} & -\frac{(k+h)}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix}}_{A} . \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{m} \end{bmatrix}}_{B} .F_A + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{h}{m} \end{bmatrix}}_{W} .x_r \\ \underbrace{\begin{bmatrix} y_1 = x_M \\ y_2 = x_m \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} . \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} \end{cases} \quad (3)$$

This model is for the active 1/4 vehicle suspension. In the models of passive and semi-active suspensions, the absence of the active damper FA and its input matrix B is noted. This model is used for designing an optimal control law to control the damper or the active actuator. The control law is described in the following section. This law should ensure passenger comfort and safety by minimizing the error between the vehicle's body displacement (x_M) and the wheel displacement (xm), as well as minimizing the difference between xm and the road profiles (x_r) traveled. For this reason, a trajectory tracking control approach is chosen, using a quadratic minimization criterion such as the LQR (Linear Quadratic Regulator).

3. Linear Quadratic Regulator (LQR)

The Linear Quadratic Regulator (LQR) is an optimal controller designed based on the state-space model of a linear system, characterized by the state matrix A, the input matrix (or vector) B, and positive weighting matrices Q and R. The objective of the LQR is to minimize the energy criterion given by (Sinha, 2007):

$$J = \frac{1}{2}\int_0^{\infty} \left(x^T Q x + u^T R u\right) dt \quad (4)$$

This criterion allows finding the solution to the algebraic Riccati equation, P:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (5)$$

This solution is used to calculate the state feedback gain for the control input, FA.

$$F_A = u = -K.x = -\left(R^{-1}B^T P\right)x \quad (6)$$

The LQR controller is among the most widely used controllers in linear systems due to its simplicity of implementation, effectiveness, and robustness. It is also used in conjunction with observers or stochastic filters, known as LQG (Linear Quadratic Gaussian), in cases where the system state is not directly measurable (Heij *et al.*, 2007). There are numerous applications of LQR control and regulation in the literature, such as active suspension systems (Louam, 1996), DC motors (Ruderman *et al.*, 2008), aircraft trajectory tracking (Lichota *et al.*, 2020), and others. Unfortunately, the main challenge lies in the selection of weighting matrices, which are typically chosen through trial and error. Research has been conducted to address this issue and explore adjustment techniques, including conventional adjustment methods (Rajan *et al.*, 2021) (e.g., dynamic adaptation for nonlinear control of an inverted pendulum on a cart) as well as hybrid approaches with artificial intelligence (Sáez & Cipriano, 1998) and metaheuristic optimization algorithms such as PSO (Mobayen *et al.*, 2011), Firefly Algorithm (Pal *et al.*, 2015), Genetic Algorithm (Wongsathan *et al.*, 2009), among others. In the following section, we will explore the use of the Firefly Algorithm for the adjustment of weighting matrices in an LQR controller applied to a 1/4 vehicle active suspension system.

4. Firefly Algorithm

The Firefly Algorithm (FA), also known as the Firefly Optimization Algorithm, is a metaheuristic algorithm inspired by the behavior of fireflies in nature. It was initially proposed and applied by Xin-She Yang (Yang X., 2008) and has since been improved and hybridized with other metaheuristic techniques (Arora & Singh, 2013) (Ab Talib *et al*., 2015). The algorithm has been applied to various optimization problems, including parameter selection for systems (Mahmoodabadi *et al*., 2018) (OLIVEIRA de *et al*., 2020) and controller parameter tuning (Pal *et al*., 2015) (Trivedi *et al*., 2016) (Ravari *et al*., 2019) (ABUT *et al*., 2019) (Setyowati *et al*., 2020) (Jadhav Vilas *et al*., 2021).

The Firefly Algorithm simulates the behavior of fireflies in search of food. Fireflies emit flashes of light to attract insects and communicate with each other. This behavior is leveraged to solve optimization problems. In the Firefly Optimization Algorithm, the following assumptions (necessary conditions) are made:

- All fireflies are unisex.
- Attraction between fireflies is proportional to their brightness.
- Brightness is determined based on an objective function.

These assumptions define the movement of fireflies within the optimization problem. The relationship governing this movement is as follows:

$$x_i^{t+1} = x_i^t + \beta.\left(x_j^t - x_i^t\right) + \alpha * (rand - 0.5) \tag{7}$$

With:

- x_i^{t+1} is the location of firefly i at generation t+1,
- x_i^t is the location of firefly i at generation t,
- x_j^t is the location of firefly j at generation t,
- β is the attractiveness coefficient, avec $\beta(r) = \beta_0 e^{-\gamma r^2}$,
- r is the distance between two adjacent fireflies, $r_{i,j} = \|x_i - x_j\|$,
- α is the randomization coefficient between [0, 1].

In this context, the parameters of the weighting matrices Q and R are defined as fireflies, representing the objectives for determining the best solutions, following the algorithm proposed by Yang (Yang X., 2008).

5. Simulations and interpretations

The implementation of the active controller for the suspension, based on the LQR regulator, is based on the following steps:

- The model of the active ¼ vehicle suspension is introduced in the Matlab/Simulink interface.
- The firefly algorithm is implemented in a Matlab program, with the definition of several parameters: *i=3* (number of objectives), *n=50* (number of fireflies), and a maximum number of iterations set to *50*. The evaluation function used is the Integral of Absolute Error (IAE), defined as follows:

$$IAE = \int e^2(t)dt \tag{8}$$

Where '*e(t)*' is the error between the road disturbance 'x_r' and the vertical displacement 'x_M'.

- The road disturbance 'x_r' is selected from various signals: impulse signal, sinusoidal signal and a graded unit step signal.

- The suspension parameters are chosen as follows (Ref): $M = 35\ kg$ (mass of the vehicle), $m = 3\ kg$ (mass of the quarter vehicle), $h = 80000$ (spring stiffness), $k = 3000$ (spring constant), $d = 250$ (damping coefficient).

These parameters are used to define the characteristics of the suspension system and will be utilized in the subsequent steps of the control design process.

(a). Impulse signal

The first signal used as input for road profiles is an impulse signal, representing unexpected road disturbances (Figure 2).

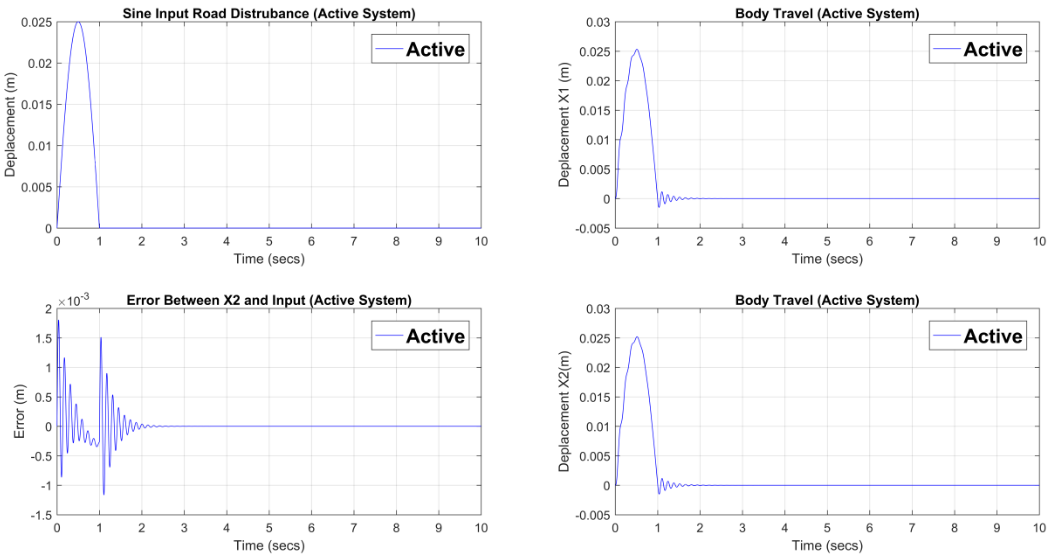

Figure 2 - Response of the active suspension to a sudden road input

This figure demonstrates that the vehicle body, x_M, and the wheel displacement, x_m, follow the proposed impulse input, as evidenced by the negligible error, on the order of 10^{-3}, as shown in the figure.

(b). Sinusoidal signal

The second signal is chosen as a sinusoidal signal, which results in a small fluctuation around *0*, also considered negligible, given their magnitude of 10^{-3} (Figure 3).

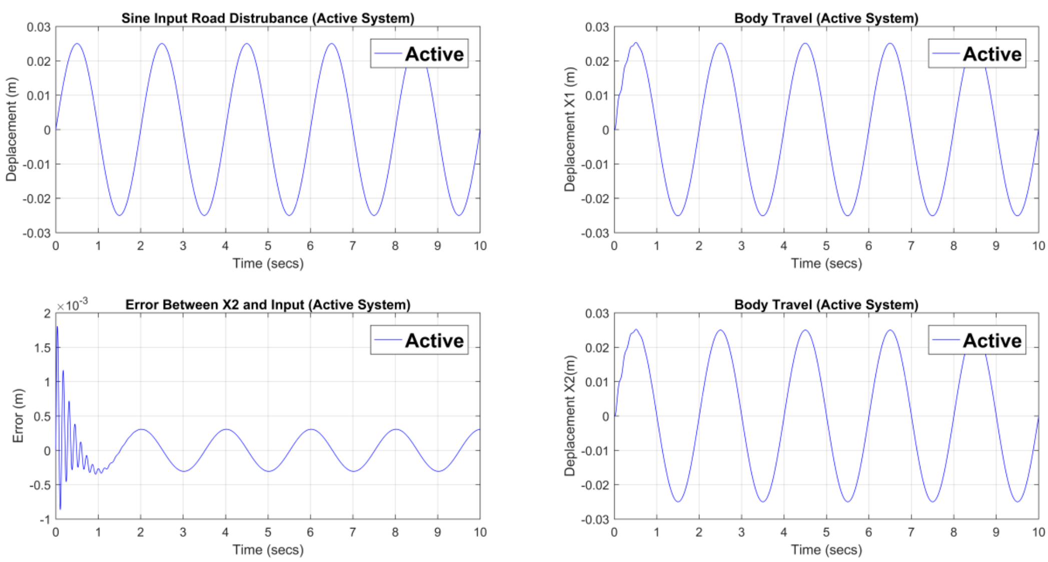

Figure 3 - Response of the active suspension to a sinusoidal input

(c). Step-wise signal

Figure 4 - Response of the active suspension to a graded unit step input, demonstrating perfect trajectory tracking and an error on the order of 0.005.

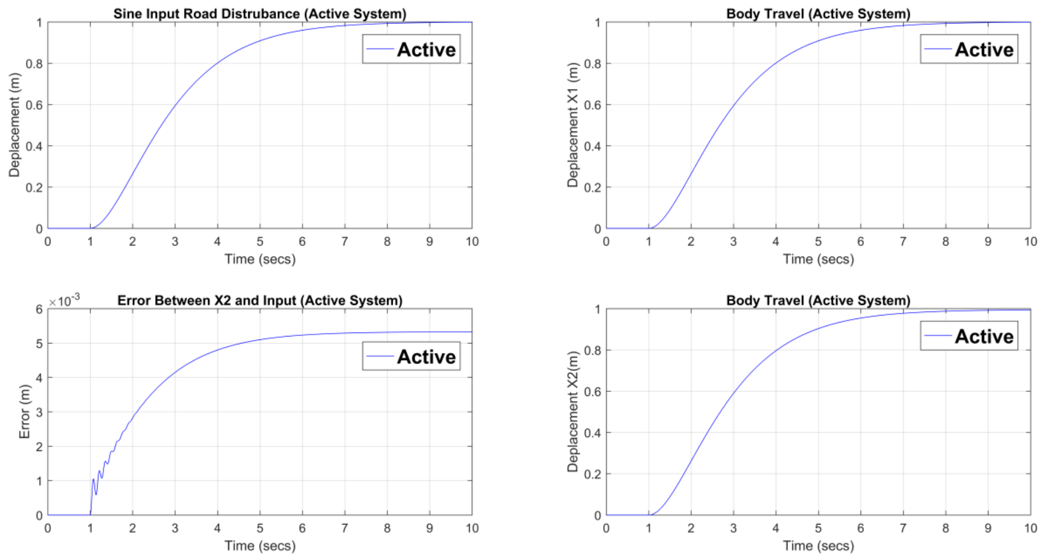

Figure 4 - Response of the active suspension to a graded unit step input

In this regard, we observe that our controller performs well with different road input signals, ensuring our objective of comfort and safety, thanks to the effective optimization of the weighting matrices Q and R using the proposed Firefly algorithm. This result appears to be superior to other control methods such as PID (Al-Khazraji, 2022), Fuzzy PID (AHMED, *et al*., 2022) employing alternative metaheuristic methods, or conventional LQR (Kumar *et al*., 2006) (Jibril & Tadese, 2020) found in the literature.

6. Conclusion et perspectives

This study investigates and simulates the design of a Linear Quadratic Regulator (LQR) for a quarter vehicle active suspension system. The LQR controller is designed to optimize the weighting matrices Q and R, which play a crucial role in achieving desired system performance. In this study, the optimization process for determining the optimal values of Q and R is carried out using the Firefly algorithm, a metaheuristic optimization technique known for its effectiveness in solving complex optimization problems.

By employing the LQR control strategy, the objective is to enhance the overall performance of the active suspension system. The weighting matrices Q and R are carefully selected to appropriately balance the trade-off between ride comfort and vehicle stability. The matrix Q represents the importance of tracking the desired states, while the matrix R reflects the control effort penalty.

Through simulations, the performance of the LQR controller with optimized Q and R matrices is evaluated. The simulations consider various road profiles and input signals that mimic real-world road conditions and disturbances. By assessing the response of the active suspension system under different scenarios, the effectiveness of the LQR controller and its ability to provide improved ride comfort and enhanced passenger safety are analyzed and interpreted.

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