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Comparative Analysis of Type-1 and Type-2 Fuzzy Controllers: Exploiting Synergies for Improved Control System Performance

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Abstract

The utilization of fuzzy controllers in control systems has gained significant attention in various fields due to their ability to handle complex and uncertain systems. This paper focuses on comparing the performance of two types of fuzzy controllers: type-1 fuzzy controller and type-2 fuzzy controller. While type-1 fuzzy controllers have been widely studied and applied, the advantages of type-2 fuzzy controllers in dealing with uncertain and ill-defined systems have garnered attention. A thorough analysis and comparison are conducted to investigate the practical benefits of type-2 fuzzy controllers. The study includes an overview of type-1 and type-2 fuzzy sets, followed by the design methodology for the two controllers. The designed controllers are implemented practically, and the results are analyzed and discussed. Previous research studies have shown the potential of type-2 fuzzy controllers to achieve superior performance compared to type-1 controllers. This paper contributes to the existing body of knowledge by presenting empirical evidence and practical insights into the performance of type-2 fuzzy controllers in real-world applications. The findings shed light on the benefits and limitations of utilizing type-2 fuzzy controllers, particularly in scenarios where system knowledge is limited or incomplete. The results of this study provide valuable guidance for control system designers and practitioners in selecting the most appropriate controller for their specific applications. Furthermore, the insights gained from this research contribute to the ongoing exploration and development of fuzzy control systems, enhancing their effectiveness in various domains.

Keywords: fuzzy controllers, type-2 fuzzy sets, Footprint of Uncertainty (FOU)

1. Introduction

The utilization of control systems in industrial settings has become increasingly crucial due to the rapid advancement of technology worldwide. The central component of a control system is the controller, often regarded as the system's brain. Various types of controllers exist, some of which require a mathematical model of the process for effective design, while others do not. The ability to design controllers without relying on mathematical models confers a significant advantage, as mathematical modeling is often a challenging step in control system design.

Among the diverse range of controllers available, fuzzy controllers have shown great promise since the introduction of fuzzy sets theory by Lofti A. Zadeh in 1965 (Zadeh, Fuzzy Sets, 1965). Initially, Zadeh's fuzzy sets were commonly called fuzzy sets until 1975, when Zadeh introduced the concept of type-2 fuzzy systems. This new approach offered additional degrees of freedom in design (Zadeh, 1975).

Fuzzy sets and fuzzy controllers have applications in numerous fields, including medicine, biology, finance, artificial intelligence, and process control. However, based on type-1 and type-2 fuzzy sets, there are two distinct types of fuzzy controllers: type-1 fuzzy controllers and type-2 fuzzy controllers.

Evidently, type-2 fuzzy sets are more intricate than type-1 and are employed more extensively. Consequently, a question arises: does utilize a type-2 fuzzy controller instead of the traditional type-1 fuzzy controller offer any practical advantages?

Some applications have demonstrated the effectiveness of type-2 fuzzy controllers. For instance, in a study by Zhou, Ying, and Zhang (2019), the authors examined the influence of the Footprint of Uncertainty (FOU) on the analytical structure of a type-2 fuzzy controller. They asserted that increasing the FOU leads the type-2 fuzzy controller to approximate a constant controller when employing Mamdani inference. At the same time, it resembles a piecewise linear controller when using Takagi-Sugeno inference. Similarly, Shi (2020) proposed a fractional order type-2 fuzzy PID controller for an inverted pendulum system. The results indicated that the type-2 fuzzy PID controller outperformed other controllers by reducing overshoot, enhancing system response speed, and efficiently handling external disturbances. Numerous other applications and studies exploring type-2 fuzzy controllers can be found in works such as Fu, Lam, Liu, Zhou, and Zhong (2021), Yang, Niu, and Lam (2022), and Sahin and Ulu (2023), among others.

Therefore, the primary objective of this paper is to design and compare two controllers for the same plant: a type-1 fuzzy controller and a type-2 fuzzy controller. By exploiting the synergies and differences between them, we aim to provide an answer to the previous question.

The structure of this paper is as follows: Section II provides an overview of type-1 and type-2 fuzzy sets. Section III outlines the methodology employed for designing the two controllers and presents the proposed procedure for their comparison. The practical application of the controllers is discussed, and the obtained results are shown in Section IV. Finally, Section V offers concluding remarks.

2. Type-1 and Type-2 fuzzy sets

In 1965, Lofti A. Zadeh introduced a groundbreaking theory known as the fuzzy logic system, which has applications in various domains, including engineering, medicine, and finance (Zadeh, 1965). The essence of a fuzzy system can be elucidated through its three distinct blocks, as depicted in Figure 1.

In this system, each input is characterized by linguistic variables, and their representations are established using membership functions with defined intervals. For instance, in Figure 1, the crisp value of 1 exhibits a similarity degree of approximately 0.65 concerning the linguistic variable N and about 0.4 concerning the linguistic variable P. Notably, each linguistic variable is associated with a single similarity degree.

The rule system governing the fuzzy logic system is encapsulated within the inference block, which follows the " **If** *Antecedent*, **then** *Consequent*" structure. At this juncture, the Consequent can take the form of a membership function, as employed in the Mamdani inference method (Mamdani, 1977), or a linear function, as utilized in the Takagi-Sugeno inference method (Takagi & Sugeno, 1985).

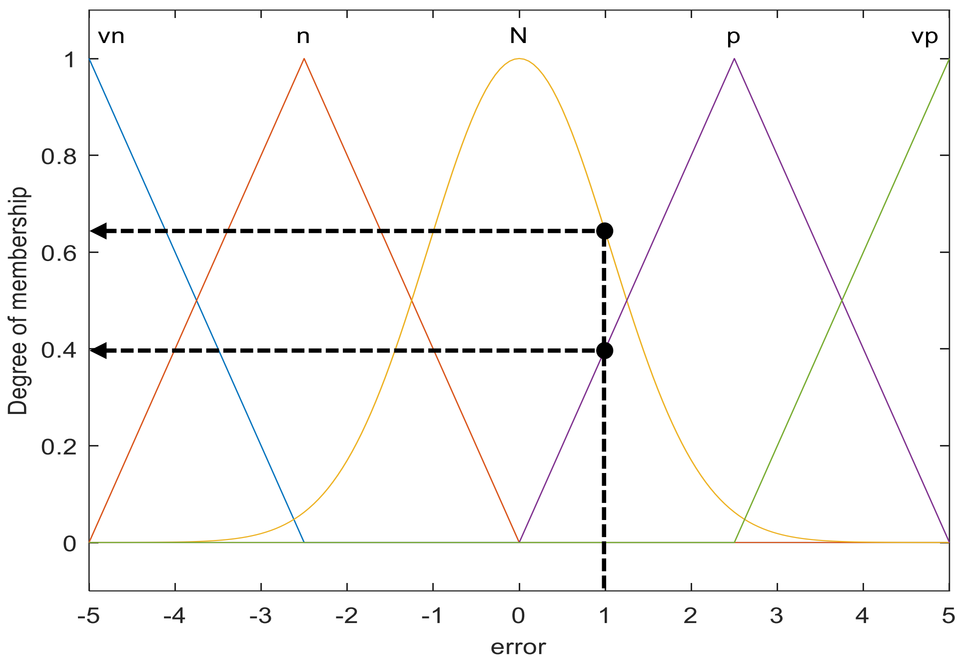

Figure 1: Membership Functions- input error

In the context of fuzzy sets, a fuzzy set of a universe of objects X is defined by a function μ that maps X into the interval [0, 1]. When X is a finite set $x_1, \dots \dots, x_n$, a fuzzy set F on X can be represented as $= (x, \mu_F(x) | x \in X)$, where $\mu_F(x)$ denotes the membership grade of x in F, which is determined using a membership function.

However, approximately a decade later, in 1975, Lofti A. Zadeh introduced type-2 fuzzy sets, necessitating the differentiation between the fuzzy sets introduced in 1965 and the newly introduced type-2 fuzzy sets. Consequently, the fuzzy sets presented in 1965 were subsequently termed type-1 fuzzy sets.

Type-2 fuzzy sets possess two membership functions for each linguistic variable. Therefore, this fuzzy set category introduces an additional imprecision level compared to type-1 fuzzy sets, where an actual value is associated with an interval of similarity degrees, as illustrated in Figure 2.

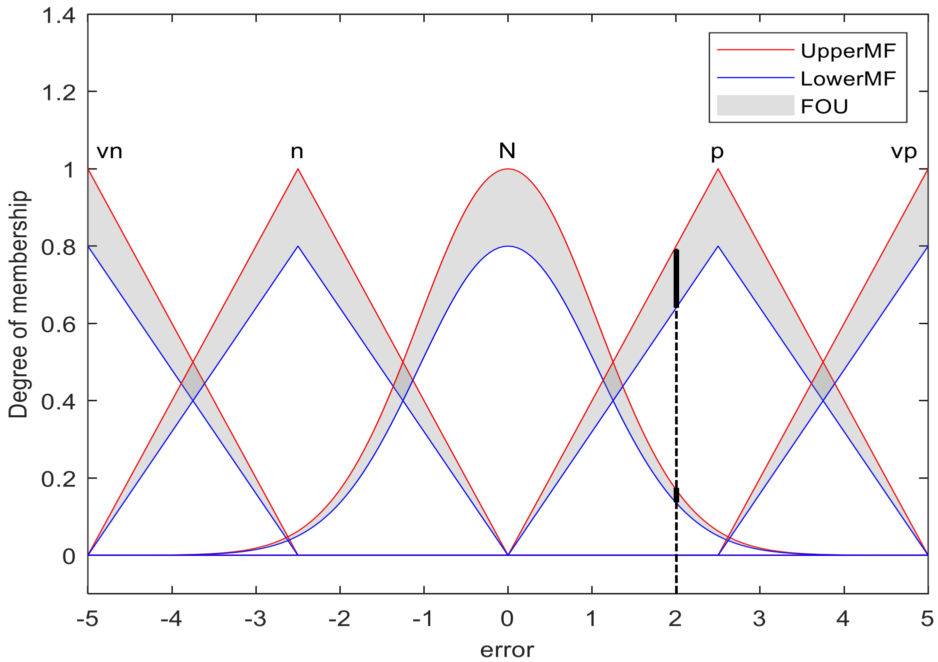

Figure 2: Membership functions type-2 fuzzy

To illustrate, in Figure 2, the crisp value of 2 exhibits an interval of similarity degrees concerning the linguistic variables N and P. This observation highlights the inherent uncertainty associated with similarity degrees. Two membership functions are linked to each linguistic variable featuring a distinctive hatched area emerges, known as the FOU (Footprint of Uncertainty). The FOU delineates the uncertainty about the similarity degrees (Mendel, Type-2 fuzzy sets and systems: An overview, 2007).

By definition, a type-2 fuzzy set $\tilde{F}$ is represented by a type-2 membership function $\mu_{\tilde{F}}(x,u)$, where $x \in X, \forall\, u \in J_x \subseteq [0;1], i.e.$, (Mendel, John, & Liu, 2006)

$$\tilde{F} = \{((x,u), \mu_{\tilde{F}}(x,u)) \mid \forall\, x \in X, \forall\, u \in J_x \subseteq [0;1]\} \quad (1)$$

The primary distinction between type-2 and type-1 fuzzy sets lies in the concept of FOU, which introduces uncertainty regarding similarity degrees. A crisp value corresponds to a range of similarity degrees associated with a specific linguistic variable (Zhou, Ying, & Zhang, 2019).

Despite this dissimilarity, the structure of the rules governing both types of fuzzy sets remains unchanged:

i) Mamdani inference method: $If\ x_1\ is\ F_1\ and\ x_2\ is\ F_2\ then\ y = B_k$
ii) Takagi-Sugeno inference method: $If\ x_1\ is\ A_1\ and\ x_2\ is\ A_2\ then\ y = g(x_1,\, x_2)$

where, $x_1\ and\ x_2$ are inputs, $F_1\ and\ F_2\ and\ B_k$ are type 2- fuzzy sets.

However, the critical question arises: What impact does the FOU, which represents the fundamental distinction between these two types of fuzzy sets, have on the dynamics of a fuzzy control system?

This study aims to address this question by designing and practically implementing both type-1 and type-2 fuzzy controllers in a position control system using a technique known as data acquisition. Through this investigation, valuable insights can be gained into the effects and implications of the FOU in fuzzy control systems.

3. Fuzzy Controller design

This study aims to design two fuzzy controllers using the Takagi-Sugeno inference method: a type-2 fuzzy controller and a type-1 fuzzy controller. Initially, a type-1 fuzzy controller was developed based on the mathematical model of the plant.

Subsequently, a type-2 fuzzy controller was designed, and both controllers were implemented in a position control system.

The initial configuration of the fuzzy system involved defining all membership functions and rules using type-1 fuzzy sets, as depicted in Figure 3.

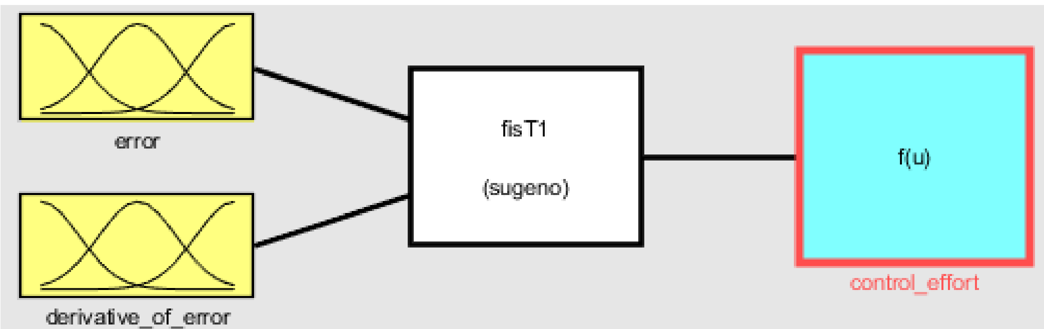

Figure 3: Fuzzy Inference System.

Simulations were conducted to analyze the system, and subsequently, the controllers were implemented in a real plant using the data acquisition technique. It is worth noting that during this implementation, the rules had to be adjusted to account for nonlinearities and noise acting in the plant. Figure 4 illustrates the system configuration utilized in the practical implementation.

The obtained type-1 fuzzy controller was then converted into a type-2 fuzzy controller using the "convertToType2" command in Matlab. All rules and membership functions were retained for this conversion as defined for the type-1 fuzzy controller. The only difference between the type-1 and type-2 controllers lies in the FOU, which was modified to investigate its influence on the system's dynamics.

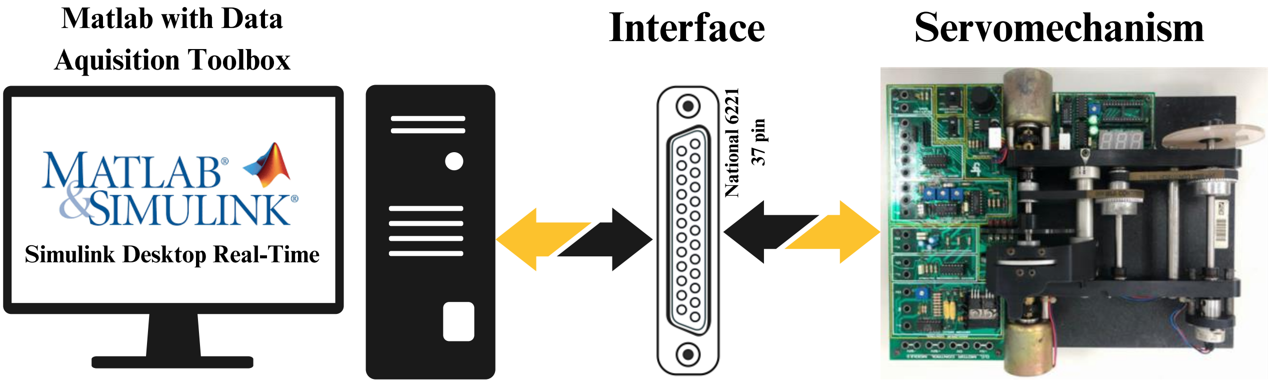

Figure 4: Scheme of the real system

The investigation of varying the FOU in the type-2 fuzzy controller yielded significant outcomes compared to the type-1 fuzzy controller.

4. Results

The results presented in this paper are based on a practical application involving a real plant. However, before implementing the controller with the real plant, it was initially obtained based on the mathematical model of the plant. Due to uncertainties, noise, and nonlinearities encountered in the real plant, some rules and membership functions had to be adjusted to optimize performance. The procedure involved three steps:

i) After refining the rules and membership functions, the type-1 fuzzy controller was applied to the position control system, and the results are depicted in Figure 5.

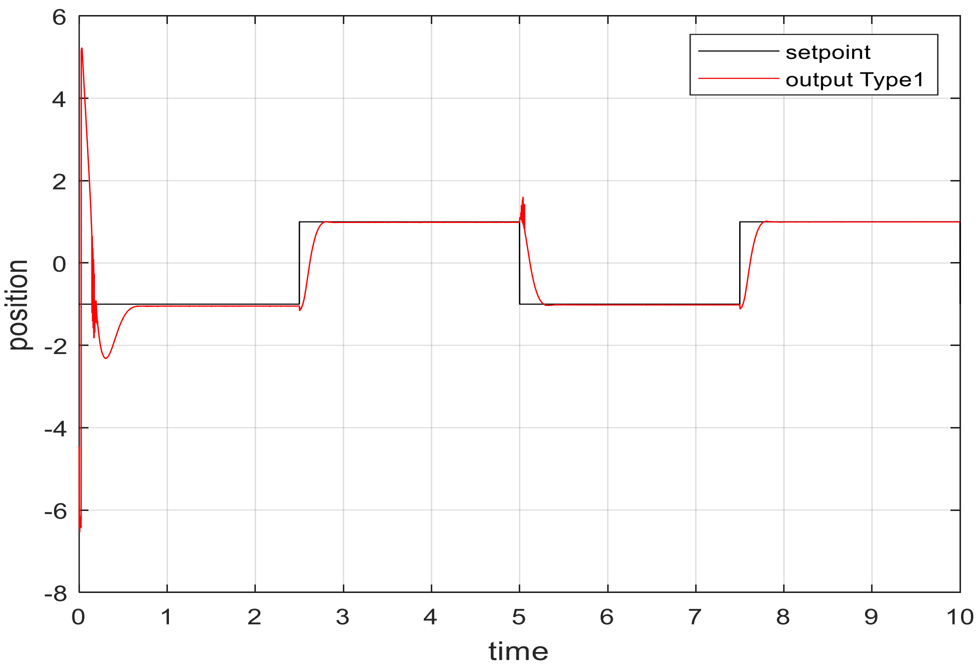

Figure 5: Response to the step of the type-1 fuzzy system

The response was fast, with a settling time of approximately 0.5 seconds, zero overshoot, and minor errors. This scenario demonstrates the effectiveness of the type-1 fuzzy controller, with the defined rules and membership functions being deemed suitable.

ii) In this step, the rules were intentionally modified to produce a poor response using the type-1 fuzzy controller. The resulting response, shown in Figure 6, indicates that the rules in their current form are far from optimal. It implies that significant changes would be required if one persists with the type-1 fuzzy controller. The figure demonstrates an unsatisfactory response despite employing the same rules and membership functions, highlighting the limitations of the type-1 fuzzy controller.

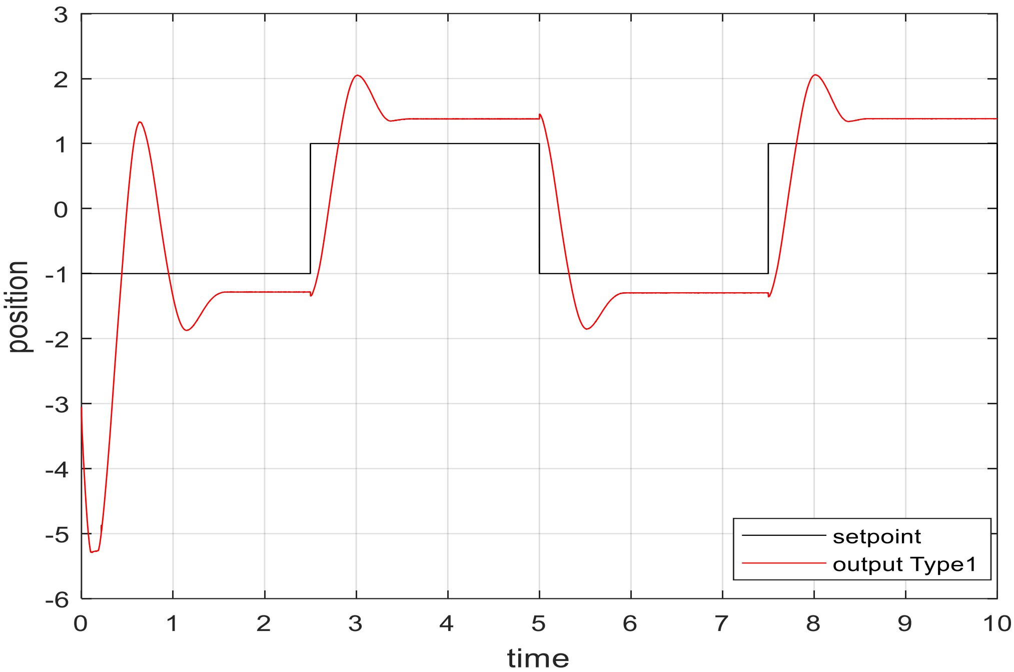

Figure 6: Response to the step type-1 fuzzy with bad rules

In this scenario, despite using the same rules and membership functions, multiple attempts were made to adjust the FOU of the type-2 fuzzy controller to achieve a better response. However, all these trials proved unsuccessful. Figure 7 provides an example of one such attempt.

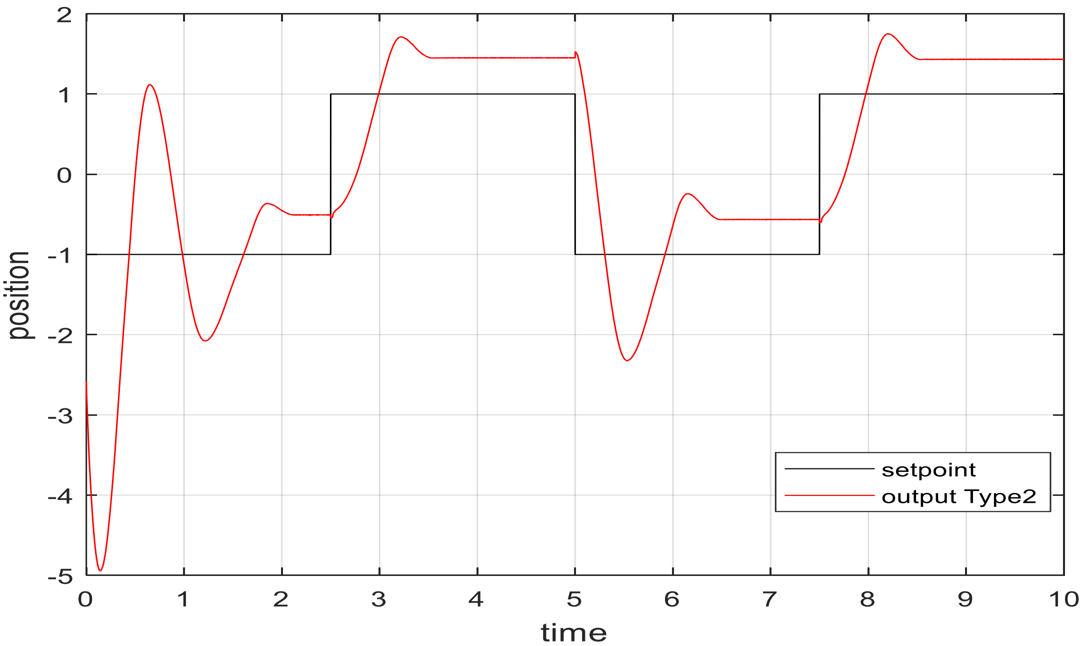

Figure 7: Response to the step type-2 fuzzy with bad rules

As depicted in the figure, it is evident that varying the FOU did not significantly improve the system's response.

iii) In the third and final step, the type-1 fuzzy controller was employed intentionally with modified rules to generate a response with a significant error. Moreover, the system was subjected to an external disturbance, resulting in an increased steady-state error. In this case, the rules can be considered a suboptimal rule set, which would require substantial modifications if one persisted in using the type-1 fuzzy controller. These modifications could include altering the membership functions, adjusting the rules, or changing the interval of the membership functions. However, it is essential to note that implementing such changes takes time and may not guarantee an optimal response.

In contrast, by maintaining all the rules and membership functions used in the type-1 fuzzy controller, a type-2 fuzzy controller was obtained. After adjusting the FOU, the response exhibited a reduced steady-state error, even in disturbances. These outcomes are illustrated in Figure 8.

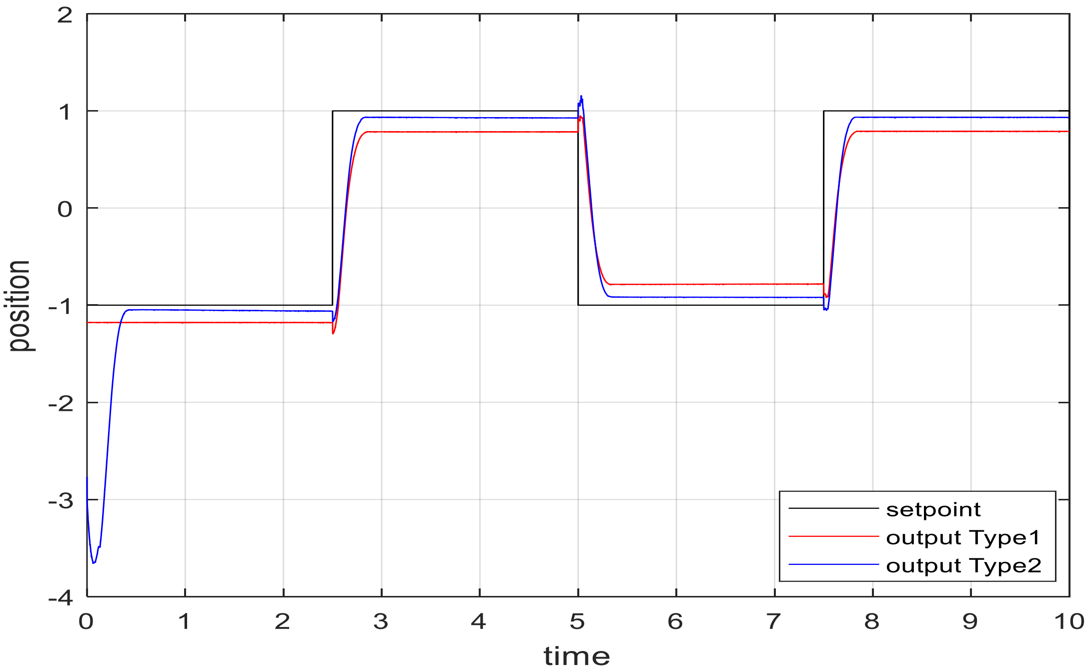

Figure 8: type-2 fuzzy, keeping all the rules and membership functions used in the type-1 fuzzy controller.

The response achieved with the type-2 fuzzy controller surpassed that of the type-1 fuzzy controller. It demonstrated higher precision and greater robustness in dealing with external disturbances. It is worth noting that the type-1 fuzzy controller yielded a response with significant errors in the presence of external disturbances, even having reasonably good rules and membership functions. However, employing the type-2 fuzzy controller while retaining the same rules and membership functions as the type-1 fuzzy system improved the system's response regarding error reduction and disturbance rejection.

5. Concusions

This study aimed to investigate the impact of the FOU on the dynamic response of a position control system. As is widely acknowledged, the effectiveness of rules and functions relies heavily on the extent of knowledge about the plant, which can be acquired through expert input or comprehensive system analysis. Initially, a type-1 fuzzy controller was designed with well-defined rules and membership functions, resulting in an optimal response with zero error and no overshoot. Subsequently, the rules were altered to elicit a poor response that could not be improved by substituting the type-1 fuzzy controller with a type-2 one. Furthermore, the rules were modified to induce a response with a significant error in the presence of external disturbances. In these latter scenarios, extensive adjustments were made to the FOU upon transitioning to the type-2 fuzzy controller to achieve an enhanced response. The findings revealed that:

- In instances where the rules set and/or membership functions were inadequately defined, despite multiple adjustments to the FOU, there was a lack of improvement in response performance. It highlights the limitations of a fuzzy control system plagued by design deficiencies stemming from ill-defined rules and/or membership functions.
- Conversely, when the rules and/or membership functions were relatively well-defined, subtle modifications to the FOU yielded a notably enhanced response, demonstrating greater resilience in external disturbances.
- The first case underscores that if the rules and membership functions are well-defined, there is no imperative need to replace the type-1 fuzzy controller with a type-2 fuzzy controller unless robustness is a critical requirement in the application. Nevertheless, it is worth noting that in all cases, the type-2 fuzzy controller exhibited superior robustness compared to the type-1 fuzzy controller, elevating the system's response to external perturbations.

Therefore, the obtained results demonstrate that utilizing a type-2 fuzzy controller can significantly enhance the system response in situations where the rules and membership functions are not well-defined, primarily due to a lack of knowledge on the part of the designer. The ability of the type-2 fuzzy controller to incorporate and effectively handle uncertainty through the FOU enables it to compensate for deficiencies in rule definition and membership function design. It highlights the importance of employing type-2 fuzzy controllers in scenarios where the designer's knowledge is limited or incomplete, as they provide a more robust and reliable solution for achieving improved system performance.

The future investigation could address the following topics:

Ill-defined rules and membership functions often appear. Future research can explore advanced techniques for rule generation and membership function optimization. It could incorporate machine learning algorithms or data-driven approaches to automatically determine the optimal rule set and membership functions based on system data and expert knowledge.

Investigating the development of adaptive type-2 fuzzy controllers could be beneficial. These controllers would dynamically adjust the FOU and other parameters based on real-time system feedback, allowing the controller to adapt and optimize its performance in changing operating conditions and uncertainties.

Conducting comparative studies between type-2 fuzzy controllers and other advanced controller types, such as neural networks, evolutionary algorithms, or model predictive control, would provide insights into their respective strengths and weaknesses in different applications. It could help identify scenarios where type-2 fuzzy controllers excel, and alternative approaches might be more suitable.

Applying type-2 fuzzy controllers to real-world industrial systems and evaluating their performance in practical settings would validate their effectiveness and provide valuable insights into their applicability, robustness, and scalability. It could involve conducting experiments, case studies, and field trials to assess the performance of type-2 fuzzy controllers in diverse domains and under varying operating conditions.

Exploring hybrid control strategies that combine the strengths of type-2 fuzzy controllers with other control techniques could be an exciting avenue for future research. Integrating type-2 fuzzy controllers with model-based or data-driven approaches, for instance, could leverage both methods' advantages and yield even better control performance and robustness.

By addressing these areas in future works, researchers can further advance the understanding and application of type-2 fuzzy controllers, improving their performance, expanding their range of applications, and contributing to control systems design and optimization.

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