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Investigation of the Effects of Buildup Factors on Electromagnetic Radiation Dose

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Abstract

The buildup factor is an important element in radiation protection and shielding. It is also an essential component of the equation for dose calculation. In this study, dose conversion factors were calculated for outside exposures from gamma-rays. The calculations were established on the point-kernel integration method where two expressions for buildup factor were tested; (a) Taylor's buildup factor and (b) Linear buildup factor. Dose calculations were performed for the buildup factors expressions at energy range of 0.01MeV – 10.00MeV. The calculations yielded two results for the Dose conversion factor, which are at variance within the range 0.01-3.00MeV and > 5.00MeV. However, there is apparent agreement between the two results for energy range 3.00-5.00MeV. The result for the Taylor's build-up factor correlates closely with the earlier results obtained from experimental and theoretical approaches. It has therefore been revealed that the choice of buildup factor used in dose calculation affects the output of the calculation.

Keywords: Build-up factor. Dose conversion factor. Electromagnetic radiation

Nomenclature

DCF	Dose rate conversion factor
ρ_m	Medium density
$P_i(E_i)$	Emission probability
μ_a	Linear attenuation coefficients of air
μ_m	Linear attenuation coefficients of medium
B_i	Dose build-up factor
r	Total distance from the source to the receptor
K	Unit normalization constant
v	Volume of the medium

1. Introduction

During the process of attenuation, light particles undergo various interaction mechanisms, including but not limited to the photoelectric effect, pair production, and Compton scattering. These interactions give rise to secondary photons, which possess an inherently indefinite potential to reach a designated target or point of interest (Akyildirim *et al.*, 2017; Gwayisa, 2022; Luggar, 1994; Sibiya, 2010). The extent to which these secondary photons augment the photon fluence at a specific dose point is often quantified using the concept known as "buildup." The buildup factor is instrumental in refining the dosimetric analysis by incorporating the contributions of scattered and secondary radiation into the primary estimation of uncollided photon flux, thus offering a more comprehensive assessment of radiation exposure (de Martino *et al.*, 2021; Mishra & Selvam, 2023; Reynoso, 2021; Chen, 1991; Verhaegen, 2016). This factor serves as a corrective measure that adjusts the initial calculation of uncollided photons to account for the additional radiation contributions, thereby ensuring an accurate representation of the cumulative dose. The methodology to quantify this adjustment is critical in radiation dosimetry, particularly in the precise evaluation of radiation protection measures, treatment planning in radiotherapy, and nuclear medicine diagnostics.

A variety of mathematical models have been developed to calculate the buildup factor, reflecting its significance in the field of radiation physics and engineering. Among these, certain formulations have gained prominence and are frequently employed due to their proven efficacy and reliability in diverse applications (Ahmed & Giddens, 1984). These models are foundational to the theoretical and practical understanding of radiation transport and its interactions within different media, providing essential insights for both academic research and clinical applications. The selection of an appropriate buildup factor model is contingent.

Linear Formula:

$$B(x) = 1 + \propto (E, Z) . x \tag{1}$$

Berger's Formula:

$$B(x) = 1 + C(E, Z) x e^{D(E,Z) \propto} \tag{2}$$

Capo's Formula:

$$B(E, x) = \sum_{i=0}^{3} \beta_i x^i \tag{3}$$

where

$$\beta_i = \sum_{j=0}^{4} C_{ij} \left(\frac{1}{E}\right)^j \tag{4}$$

Taylor's Formula:

$$B(x) = A e^{-\propto_1 (E,Z) \mu x} + (1 - A) e^{-\alpha_2 (E,Z) \mu x} \tag{5}$$

Where $A, \propto_1, \propto_2, C,$ are constants, μ is the linear attenuation coefficient of photons of i^{th} energy group, x is the distance travelled by the photons (Akyildirim *et al.*, 2017)

2. Method of calculation

The dose rate conversion factor DCF(r) converts activity concentration of a homogenous medium of density ρ_m (g/cm^3) to dose rate (Kocher & Sjoreen, 1985; Mustapha *et al.*, 1999). When we choose a point in air, DCF(r) is given approximately by

$$DCF(r) = \frac{K}{4\pi}\sum \rho_m P_i(E_i)E_i \frac{\mu(E_i)}{\rho_a}\int \frac{1}{r^2}B_i(\mu_m r_m)\, e^{-\mu_a r_a}e^{-\mu_m r_m}dv \tag{6}$$

Where $P_i(E_i)$ is the emission probability i.e. number of gamma rays with energy Ei emitted per unit time per primary disintegration $(s^{-1}Bq^{-1})$, where $B_i(\mu_m r_m)$ is the dose build-up factor for the medium, v is the volume of the medium, μ_a and μ_m are the linear attenuation coefficients of photons (cm) in air and in the source medium respectively, r is the total distance from the source to the receptor (cm), r_a and r_m are the distances travelled by photons (cm) through air and the source medium respectively, ρ_a is air density, ρ_m is the medium density and K is a unit normalization constant which equals 5.04 x 10^{-3} (GygMeV^{-1}sy^{-1}). The build-up expressions used in this calculation are of the form given in Equations (1) and (6).

The volume element dv is transformed into spherical coordinates $(r, \theta, \emptyset)$. Substituting:
$v = r^2 sin\theta dr d\theta d\emptyset$

$$r_m = r - \frac{h}{cos\theta},$$

and

$$r_a = \frac{h}{cos\theta},$$

the buildup factors (Taylor and Linear respectively), and the limits of integration $\left[\frac{h}{cos\theta} \le r \le \infty,\ 0 \le \emptyset \le 2\pi,\ 0 \le \theta \le \frac{\pi}{2}\right]$ into Equation (6), we obtained two sets of equation defined by

$$DCF(r) = \frac{K}{4\pi}\sum \rho_i(E_i)E_i \frac{\mu(E_i)}{\rho}\int_{r_a}^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\left[\frac{1}{r^2}e^{-\mu_a\frac{h}{cos\theta}}[A_1 e^{-\alpha_1\mu_m r_m} + A_2 e^{-\mu_a r_a}e^{-\mu_m r_m}]e^{-\mu_a r_a}e^{-\mu_m r_m}r^2 sin\theta dr d\emptyset d\theta\right] \tag{7}$$

Where $C = \frac{K}{4\pi}\sum \rho_i(E_i)E_i\frac{\mu(E_i)}{\rho}$

$$DCF(r) = C\int_{r_a}^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\frac{1}{r^2}e^{-\mu_a\frac{h}{cos\theta}}[1 + \propto (\mu_m r_m)]e^{-\mu_a r_a}e^{-\mu_m r_m}r^2 sin\theta dr d\emptyset d\theta \tag{8}$$

Equation (7) is simplified further to:

$$DCF(r) = C\int_{r_a}^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\left[\begin{matrix}\frac{A_1 e^{-\mu_m(\propto_1+1)r}}{-\mu_m(\alpha_1+1)}e^{\mu_m(\propto_1+1)\frac{h}{cos\theta}} + \\ \frac{A_2 e^{-\mu_m(\alpha_2+1)r}}{-\mu_m(\alpha_2+1)}e^{\mu_m(\propto_1+1)\frac{h}{cos\theta}}\end{matrix}\right]e^{-\mu_a\frac{h}{cos\theta}}sin\theta dr d\emptyset d\theta \tag{9}$$

Integrating Equation (9) with respect to r and $\emptyset$, we obtain;

$$DCF\ (r) = \int Ke^{-\mu_a\frac{h}{cos\theta}sin\theta d\theta\left(\frac{2\pi}{\mu_m}\left(\frac{A_1}{1+\alpha_1}+\frac{A_2}{1+\alpha_2}\right)\right)} \tag{10}$$

Equation (10) can however be integrated numerically to obtain the dose rate conversion factor.

Similarly, Equation (8) is simplified to give

$$DCF(r) = C\int_{r_a}^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} e^{-\mu_a\frac{h}{cos\theta}} sin\theta \left[1+\propto \mu_m(\frac{h}{cos\theta})\right] e^{-\mu_m\left(r-\frac{h}{cos\theta}\right)} dr d\emptyset d\theta \qquad (11)$$

Integrating equation (11) with respect to r and $\emptyset$, we obtain

$$DCF(r) = \frac{2\pi C}{\mu_m}\ (1+\alpha)\int_0^{\frac{\pi}{2}} e^{-\mu_a\frac{h}{cos\theta}} sin\theta d\theta \qquad (12)$$

Equation (12) gives the dose rate conversion factor when integrated numerically.

3. Results and discussion

The data input for the numerical integration process were specified as follows: the height (h) is set at 100 cm, with the densities for air and the medium designated at 1.21 x 10^{-3} and 1.4 g cm^{-3}, respectively. Additional constants, including $\mu_a, \mu_m, \alpha_1, \alpha_2, \frac{\mu E_a}{\rho}, A_1$, and A_2, were sourced from the study conducted by Mustapha *et al*. (1999) (Table 1).

Table 1 - Constants by Mustapha *et al*. (1999)

Energy (MeV)	μ_a	μ_m	$\frac{\mu E_a}{\rho}$	A_1	A_2	α_1	α_2
0.01	0.00607	36.666	4.640	12.5	-11.5	-0.111	0.006
0.03	0.00042	1.6828	0.150	12.5	-11.5	-0.111	0.006
0.05	0.00025	0.5488	0.0403	12.5	-11.5	-0.111	0.006
0.10	0.00019	0.2492	0.0232	12.5	-11.5	-0.111	0.006
0.30	0.00013	0.1512	0.0287	12.5	-11.5	-0.111	0.006
0.50	0.00011	0.1228	0.0297	12.5	-11.5	-0.111	0.006
1.0	0.00008	0.0893	0.0279	9.9	-8.9	-0.088	0.029
10.0	0.00002	0.03234	0.0145	2.6	-1.6	-0.050	0.084

Analysis of the dose rate conversion factor for mono-energetic gamma-rays, ranging energies from 0.01 to 10 MeV (Figure 1), revealed two distinct ranges of interest. The first range, 0.01-3.00 MeV, and a second range beyond 5.00 MeV exhibited variability in the calculated factors. However, a notable concurrence was observed within the energy spectrum of 3.00-5.00 MeV.

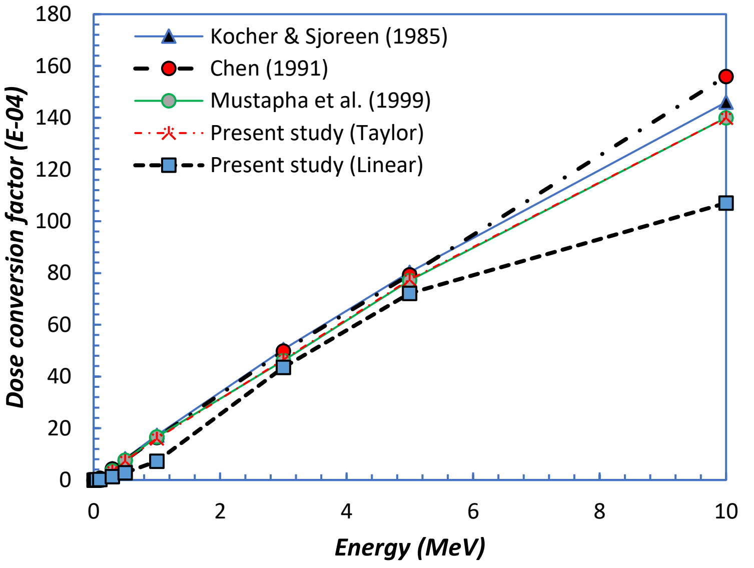

Figure 1- Relationship between the present result and earlier published results.

Additionally, the findings pertaining to the Taylor's buildup factor exhibited a significant correlation with previously established results, including those derived from experimental methodologies by Kocher & Sjoreen (1985), as well as theoretical approaches by Mustapha *et al.* (1999) and Chen (1991). This consistency emphasizes the robustness of the analytical methods employed and supports the validity of the integrated data in modeling the dose rate conversion factors across the specified energy ranges.

Conclusion

The calculation of dose rate conversion factors for gamma-rays has been conducted under two distinct scenarios: (a) employing the Taylor buildup factor and (b) utilizing the Linear buildup factor. The findings of these calculations reveal a considerable impact of the chosen buildup factor expression on the resultant dose rate conversion factor. This finding emphasizes the critical importance of selecting an appropriate buildup factor for inclusion in dose calculation processes. The selection process demands a thorough understanding of the underlying physical principles and the specific requirements of the application in question. Given the significant variance in dose rate conversion factors attributed to the differential buildup factor expressions, it is imperative for practitioners and researchers to exercise due diligence in choosing the most suitable buildup factor. This approach ensures that dose calculations are not only accurate but also reflective of the true radiation exposure levels, thereby facilitating precise risk assessment and effective radiation protection strategies.

References

Ahmed, S. A., & Giddens, D. P. (1984). Pulsatile poststenotic flow studies with laser Doppler anemometry. *Journal of Biomechanics*, *17*(9), 695–705. https://doi.org/10.1016/0021-9290(84)90123-4

Akyildirim, H., Waheed, F., Günoğlu, K., & Arkurt, I. (2017). Investigation of buildup factor in gamma-ray measurement. *Acta Physica Polonica A*, *132*(2), 1203–1206. https://bibliotekanauki.pl/articles/1031578.pdf

de Martino, F., Clemente, S., Graeff, C., Palma, G., & Cella, L. (2021). Dose Calculation Algorithms for External Radiation Therapy: An Overview for Practitioners. *Applied Sciences 2021, Vol. 11, Page 6806*, *11*(15), 6806. https://doi.org/10.3390/APP11156806

Gwayisa, T. (2022). *Shielding material selection for the disposal of radioactive waste from* ***99 Mo production***. *PhD Dissertation* (North-West University (South Africa)). North-West University (South Africa). https://repository.nwu.ac.za/handle/10394/39339

Kocher, D., & Sjoreen, A. (1985). Dose-rate conversion factors for external exposure to photon emitters in soil. *Health Physics*, *48*(2), 193–205.

Luggar, R. (1994). *Rayleigh scattered photons for substance identification*. University of Surrey (United Kingdom).

Mishra, S., & Selvam, T. P. (2023). Radiation Dosimetry. *Handbook of Metrology and Applications*, 1–26. https://doi.org/10.1007/978-981-19-1550-5_116-1

Mustapha, A., Patel, J., & Rathore, I. V. S. (1999). Assessment of human exposures to natural sources of radiation in Kenya. *Radiation Protection Dosimetry*, *82*(4), 285–292. https://doi.org/10.1093/oxfordjournals.rpd.a032637

Reynoso, F. J. (2021). Photon Beam Dosimetry of Conventional Medical Linear Accelerators. *Radiation Therapy Dosimetry: A Practical Handbook*, 254–266. https://doi.org/10.1201/9781351005388-16/

Sibiya, T. (2010). *Radiation shielding design, verification and dose distribution calculations for industrial and insect irradiation facilities*. (Doctoral dissertation, University of the Witwatersrand, Johannesburg, 1-194. https://wiredspace.wits.ac.za/

Chen, S.Y. (1991). Calculation of effective dose-equivalent responses for external exposure from residual photon emitters in soil. *Health Physics*, *60*(3), 411–426. https://journals.lww.com/health-physics/abstract/1991/03000

Verhaegen, F. (2016). Monte carlo modeling of external photon beams in radiotherapy. In *Monte Carlo Techniques in Radiation Therapy* (2nd Edition). CRC Press. https://doi.org/10.1201/9781003211846-6