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Performance Comparison of PID and LQR Control for DC Motor Speed Regulation
Comparação de desempenho do controle PID e LQR para regulação de velocidade de motor DC

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Abstract
The aim of this research paper is to provide a comprehensive performance comparison of proportional-integral-derivative (PID) and linear quadratic regulator (LQR) control methods for DC motor speed regulation. First, a mathematical description of a permanent magnet DC motor model is offered, followed by the control systems PID and LQR, and finally, findings and discussions of whether the use of both control strategies is advantageous for DC. The results demonstrate how well LQR performs under various operating situations when compared to PID control. Engineers and researchers looking to optimize control system design for DC motor speed regulation may benefit a lot from the results of this study.
Keywords: Optimal Control. LQR Regulator. PID Regulator. DC Motor. Performance.

Resumo
O objetivo deste artigo de pesquisa é fornecer uma comparação abrangente de desempenho dos métodos de controle proporcional-integral-derivativo (PID) e regulador linear quadrático (LQR) para regulação de velocidade de motor CC. Primeiro, uma descrição matemática de um modelo de motor CC de ímã permanente é oferecida, seguida pelos sistemas de controle PID e LQR e, finalmente, descobertas e discussões sobre se o uso de ambas as estratégias de controle é vantajoso para CC. Os resultados demonstram o quão bem o LQR funciona em várias situações operacionais quando comparado ao controle PID. Engenheiros e pesquisadores que buscam otimizar o projeto do sistema de controle para regulação de velocidade do motor CC podem se beneficiar muito dos resultados deste estudo.
Palavras-chave: Controle Ótimo. Regulador LQR. Regulador PID. Motor DC. Desempenho.

1. Introduction

Electric control actuators in the form of direct current (DC) motors are widely used in a number of industrial systems and products, such as robotics, automated machine tools, and electric vehicles (Krause *et al*., 2013). In DC motors, the armature reaction and the effects of external disturbances are neglected. Linear systems are frequently modelled first, followed by linear control techniques such as PID control and fuzzy control, without regard for the DC motor's high energy consumption. The control objective is to develop a controller which satisfy the parameters for the transient, steady state, and frequency domains. Using quadratic performance indices, which generally specify physical principles, the problem can be managed through using the Linear Quadratic Regulator (LQR), an optimal control approach (Bourgoies *et al*., 2007; Kizmaz, 2023).

The present work compares the performance of the Linear Quadratic Regulator (LQR) and Proportional Integral Derivative (PID) control systems in controlling the speed of a DC (Qiang, 2014; Dorf and Bishop, 2016). This research is critical because precise speed regulation of DC motors is essential in a variety of industrial and robotic applications, affecting efficiency, precision and overall system performance. The choice between LQR and PID controllers has a considerable impact on the motor's capacity to achieve and maintain desired speeds under changing load conditions and disturbances (Ogata, 2009). Understanding their comparative performance can help engineers choose the best control method for certain applications, maximizing performance and energy economy.

This research is driven by the necessity to thoroughly assess and compare the performance of LQR and PID control systems in DC motor speed regulation. While PID controllers are frequently utilized due to their simplicity and efficacy in a variety of applications, LQR controllers provide a theoretically ideal solution under specific situations but are less commonly used in industry (Dorf and Bishop, 2016; Astrom and Murray., 2008; Athans and Falb, 1966) .

2. Theoretical comparison between LQR and PID

This section provides a theoretical comparison between LQR and PID to understand the main advantages and limits of each control strategy:

2.1. Advantages and limits of LQR Control

The main advantages and limits of LQR Control systems can be summarised as follow (Kizmaz, 2023; Astrom and Murray, 2008; Athans and Falb, 1966; Lewis *et al*., 2012; Anderson and Moore, 2007):

2.1.1. Advantages

- Optimal Control: LQR controllers offer optimal control by minimizing a quadratic performance index, resulting in optimal steady-state and transient responses under specific conditions.
- LQR control often exceeds classical PID control in terms of settling time, overshoot, and disturbance rejection.
- State Feedback: Because LQR control provides for direct feedback of system states, it may control several variables more precisely at the same time, making it suited for multivariable systems.
- Robustness to Model Uncertainty: LQR controllers can withstand certain types of model uncertainty and disturbances, especially when combined with techniques such as robust control or Kalman filtering.
- LQR control is founded on strong mathematical concepts, which provide a clear foundation for analysis and design.

2.1.2. Limits

- Complexity: Creating and implementing LQR controllers can be difficult, involving comprehensive system modeling and the determination of optimal control gains using system matrices.
- Sensitivity to Model correctness: LQR control performance is largely dependent on the correctness of the system model and the quality of state estimation, which can be difficult in real-world applications.
- Nonlinear Systems: LQR control is inherently built for linear systems and may not operate effectively in nonlinear systems unless linearization or adaptation are used.
- High computational cost: Solving Riccati's equation and calculating optimal control gains can

be computationally intensive, especially for large-scale systems or in real time.

- Implementation Difficulties: Practical use of LQR control may necessitate sophisticated knowledge of control theory, numerical methodologies, and system identification techniques.

2.2. Advantages and limits of PID Control

The key advantages and limits of PID Control systems can be summarised as follow (Ogata, 2009; Dorf and Bishop, 2016; Astrom and Murray, 2008):

2.2.1. Advantages:

- Simple to develop and implement,
- Can provide robust performance under a variety of operating circumstances,
- Allows for precise steady-state tracking of setpoints or reference signals, reducing error over time.

2.2.2. Limits:

- Limited Performance for complex Systems: In highly nonlinear or complicated systems, PID controllers may struggle to attain the desired performance without substantial tuning.
- Tuning Sensitivity: Proper tuning of PID parameters (proportional, integral, and derivative gains) is critical for peak performance. Improper tuning can cause oscillations, overshoot, and sluggish response times.
- Integral Windup: When the integrator output exceeds the controller's actuation capability, PID controllers may struggle to recover from big disturbances.

3. Mathematical modeling and dynamic system

The motor transfer function can be built using a simple mathematical model of a direct current motor. The DC motor equations are divided into their mechanical and electrical components, from which the transfer function is derived, as well as their connections. The equations for the electrical portion can be obtained as follows (Krause *et al.*, 2013; Bouazza and Mouhous, 2019; Hebert *et al.*, 1985; Chateigner, 2006):

$$V(s) = Ra\ Ia + (La\ Ia)s + K\emptyset\ w \tag{1}$$

$$Ia(s) = \frac{Va - K\emptyset w}{Ra + La\ s} \tag{2}$$

$$Ea = K\emptyset w \tag{3}$$

with :

V : voltage at the motor terminal [V]
w : the motor speed [rad/s]
Ia : current of winding [A]
KØ: electromotive force constant (emf) (Vs/rad]
Ra : resistance at the terminal [Ω]
La : inductance at the terminal [H]

The equation for the mechanical component can be calculated using Newton's law, which states that the total of electrical and load torques equals the load and motor inertia multiplied by the derivative of the angular rate, as shown in equations 4-7:

$$j\frac{dw}{dt} = Te - Tl \tag{4}$$

$$j\frac{dw}{dt} = K\emptyset\ Ia - Tl - bw \tag{5}$$

$$j\ s\ w = K\emptyset\ Ia - Tl(s) - bw \tag{6}$$

$$w(s) = \frac{K\emptyset\ Ia - Tl}{js + b} \tag{7}$$

where:

J: loads and rotor inertia [kg/m2]

b: Damping viscosity [N×m×s/rad]
Tl: Loaded torque [N×m]
Te: electrical torque [N×m].

As the voltage is the input to the system and the speed is the output, the required transfer function issymbolized by w(s)/v(s). This form is derived by dividing equation (9) by equation (4). It can also be foundby drawing the block diagram of a DC motor using the same equations as shown in Figure 1.

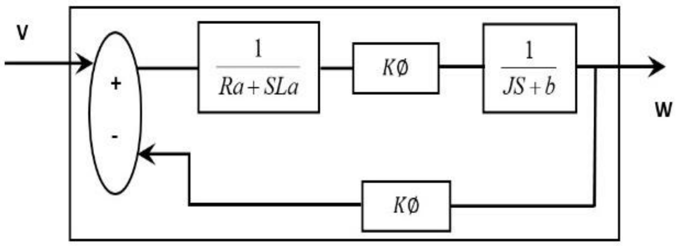

Figure 1 - Diagram of the DC motor block (plant system)

As seen in Figure 2, the schematic depicts a closed loop and a root locus analysis for a DC motor system. Consequently, the root locus analysis equation (8) can be applied:

$$\frac{C}{R} = \frac{G}{1+GH} \tag{8}$$

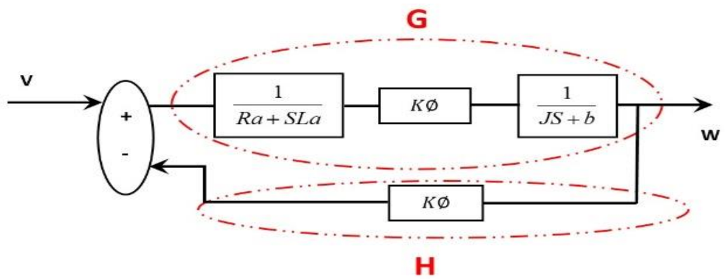

Figure 2 - Analysis of root locus in DC motor system

Equations (9) and (10) can be used to derive the transfer function for a DC motor based on Figure 2 and equation (8):

$$\frac{w(s)}{V(s)} = \frac{(\frac{K\emptyset}{Ra+La\,s})(\frac{1}{j\,s+b})}{1+K\emptyset(\frac{K\emptyset}{Ra+La\,s})(\frac{1}{j\,s+b})} \tag{9}$$

or

$$\frac{w(s)}{V(s)} = \frac{K\emptyset}{(K\emptyset)^2+(Ra+La\,s)(j\,s+b)} \tag{10}$$

Given the excitation field armature in Figure 1, the voltage can be reformed using the formulas (11–17) as follows:

$$V = Ra.Ia + La.\frac{dIa}{dt} + K\emptyset.W \tag{11}$$

$$\frac{dIa}{dt} = -\frac{Ra}{La}ia - \frac{K\emptyset}{La}w + \frac{1}{La}V \tag{12}$$

Using Newton‘s second law:

$$J\frac{dW}{dt} = Te - TL - bW \tag{13}$$

Where b is the damping friction .thus,

$$\frac{dw}{dt} = \frac{1}{J}(K\emptyset.Ia - TL - bw) \quad (14)$$

$$S = -\frac{Ra}{La}Ia(s) - \frac{K\emptyset}{La}W(s) + \frac{1}{La}V(s) \quad (15)$$

$$\left(s + \frac{Ra}{La}\right)Ia(s) = -\frac{K\emptyset}{La}W(s) + \frac{1}{La}V(s) \quad (16)$$

$$\left(s + \frac{b}{j}\right)w(s) = \frac{1}{J}K\emptyset.Ia(s) - \frac{1}{J}TL(s) \quad (17)$$

The system's input, output, and states must all be specified in order to employ a dynamic system technique. In a DC motor structure, the states are current (I) and angular rate (dW/dt), and the applied voltage (V) is the input and the angular velocity (w) is the output.

Equations (18–23) show that the system is linear; hence, the state space has the following expression:

$$\dot{x}(t) = [A]x(t) + [B]u(t) \quad (18)$$

$$y(t) = [C]x(t) + [D]u(t) \quad (19)$$

Where :

$\dot{x}(t)$ indicates the vectors of states, $y(t)$ refers to the output and $u(t)$ stands for input control signal.

$$\dot{x}(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\,(iendw) \quad (20)$$

with:

[A] = state matrix (n×n)

[B] = input matrix (n×p)

[C] = output matrix (q×n)

[D] = feed forward (zero) matrix (q×p)

Using the previously described method and the results of equations (12) and (14), the state space is as follows:

$$\frac{d}{dt}\begin{bmatrix} i \\ w \end{bmatrix} = \begin{bmatrix} -\frac{Ra}{La} & -\frac{K\emptyset}{La} \\ \frac{K\emptyset}{J} & -\frac{b}{j} \end{bmatrix}\begin{bmatrix} i \\ w \end{bmatrix} + \begin{bmatrix} \frac{1}{La} \\ 0 \end{bmatrix}\text{V} \quad (21)$$

$$\text{W} = [0\ 1]\begin{bmatrix} i \\ w \end{bmatrix} + [0]V \quad (22)$$

$$\text{A} = \begin{bmatrix} -\frac{Ra}{La} & -\frac{K\emptyset}{La} \\ \frac{K\emptyset}{J} & -\frac{b}{J} \end{bmatrix}, \text{B}=\begin{bmatrix} \frac{1}{La} \\ 0 \end{bmatrix}, \text{C}=[0\ 1], \text{D}=[0] \quad (23)$$

Consequently, Figure 3 depicts the block diagram for the dynamic system of a DC motor.

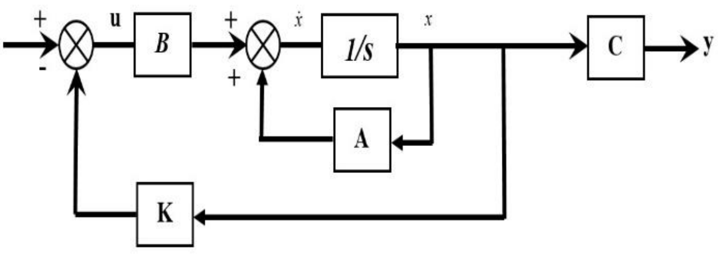

Figure 3 - Dynamic system block diagram

4. Control strategies

4.1. The LQR control approach

The LQR state feedback arrangement is shown in Figure 4 (Dorf and Bishop, 2016; Astrom and Murray, 2008). The best control system is what this design is categorized as. However, this will result in the realization of useful parts that offer the intended operational performance. As a result, the performance metrics are easily modified in terms of time. Therefore, the steady state and transient performance indexes are specified in the time domain.

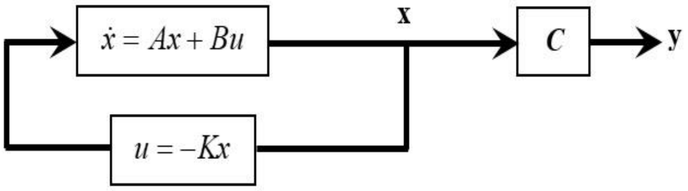

Figure 4 - LQR Regulator structure

A control system's efficiency can be expressed using integral performance measures. As a result, the system's design must prioritise reducing a performance metric, like the squared error integral. As can be seen from equations (24-25), the performance index in its specific form is:

$$j = \int_0^{tf} [(trackingerror)^2 + (weightedinputs)^2]dt \quad (24)$$

Or

$$j = \int_0^{tf=\infty} (x^T Qx + u^T Ru)dt \quad (25)$$

where R and Q stand for weighting factors and controller design parameters, respectively, and x is the state vector, x^T is the transpose of x, and tf is the final time. The design parameters of the controller and factors are chosen by trial and error, respectively. The following is the control input u [12, 14, 15]:

$$u = -Kx = [K1\ K2\ K3 \ldots \ldots Kn]x \quad (26)$$

From the system's state space, as represented by equation (27):

$$\dot{x} = \mathrm{A}x + Bu = Hu \quad (27)$$

Equations (28–31) provide the maximum value of H when it is known:

$$H = x^t Qx + u^t Ru + ʎ^T(Ax + Bu) \quad (28)$$

$$\dot{x} = Ax + Bu = \left(\frac{\partial H}{\partial ʎ}\right)^T \quad (29)$$

$$-\dot{ʎ} = \left(\frac{\partial H}{\partial ʎ}\right)^T = Qx + A^T ʎ \quad (30)$$

$$0 = \frac{\partial H}{\partial ʎ} = -Ru + ʎ^T B \quad (31)$$

Therefore:

$$u = -Ru + B^T ʎ \quad (32)$$

and equation (33) :

$$ʎ(t) = P(t)x(t) or\ ʎ = Px \quad (33)$$

we have also that equations (34 - 39)

$$u = -R^{-1}B^T Px \quad (34)$$

$$\dot{ʎ} = \dot{P}x + P\dot{x} \quad (35)$$

$$\dot{ʎ} = \dot{P}x + P(Ax - BR^{-1}B^T Px) \quad (36)$$

$$-\dot{P}x - PAx + PBR^{-1}B^T Px = Qx + A^T Px \quad (37)$$

$$-\dot{P} = PA + A^T P - PBR^{-1}B^T P + Q \quad (38)$$

$$0 = PA + A^T P - PBR^{-1}B^T P + Q \quad (39)$$

By trial and error, the design expert determines the two matrices Q and R. To keep J small, choosing Q large typically implies. To maintain J small, however, choosing R big necessitates a smaller control signal u (u). Selected as positive semi-definite and positive definite, respectively,

are Q and R. As a result, at any time t, the scalar quantities $X^T QX$ and Ru^2 are always the same—that is, they are always either positive or zero.
Equation (29) is also referred to as the Riccati Equation because it is easily coded for a computer or solved using a programming tool. Figure 4 displays the state-space configuration of the Linear Quadratic Regulator (LQR), which is considered the best control.
Combining Figures 1 and 3 results in Figure 4, which illustrates how to utilise a LQR controller with a DC motor. The schematic diagram of the DC motor under LQR control is shown in Figure 5.

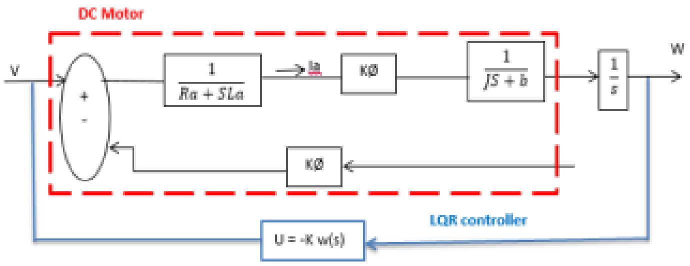

Figure 5 - Schematic diagram of the DC motor controlled by LQR

4.2. PID Controller synthesis

State variables are used in the algorithms provided under the subheadings thus far. In this situation, observable state variables are required. This is why the state observers are there. Any system's observed state variables can be estimated using a Kalman estimator, reduced order observer, or Lüenberger observer.
Since the PID controller is an output-based controller, it might not require an observer at all. Implementing feedback involves going from output to input. A reduced number of the system's measurable outputs (y) are required for the PID controller. The input signal u is a three-term controller as follows

$$\mathbf{u} = -\mathbf{K}_p(\mathbf{r}-\mathbf{y}) - \mathbf{K}_i \int_0^t (\mathbf{r}-\mathbf{y})\, dt - \mathbf{K}_d\left(\dot{r} - \dot{y}\right) \tag{40}$$

where $Kp \in \Re^{n+m}$, $Ki \in \Re^{n+m}$ and $Kd \in \Re^{n+m}$ are proportional, integral, and derivative feedback gain matrices. Considering the help of Eq. (1)-(2) and Eq. (40), Eq. (41) is obtained as follow:

$$\begin{aligned}\mathbf{u} &= \left(\mathbf{K}_p\mathbf{r} + \mathbf{K}_i \int \mathbf{r}dt + \mathbf{K}_d\dot{\mathbf{r}}\right) - \mathbf{K}_p\mathbf{Cx} - \mathbf{K}_i \int \mathbf{y}dt - \mathbf{K}_d\mathbf{C}(\mathbf{Ax}+\mathbf{Bu}) \\ &= \mathbf{u}_r - \overline{\mathrm{K}}_p\mathbf{x} - \overline{\mathrm{K}}_i \int_0^t \mathbf{y}\, dt\end{aligned} \tag{41}$$

Here the gains $\bar{\mathbf{K}}p$, $\bar{\mathbf{K}}i$ and the input residue Ur are all defined as

$$\bar{K}p = (\mathrm{Im} + \mathrm{k_d CB})^{-1}(\mathrm{k_p} + \mathrm{K_d CA}) \tag{42}$$

$$\bar{K}_i = (\mathrm{Im} + \mathrm{k_d CB})^{-1} K_i \tag{43}$$

$$u_r = K_P r + K_i \int r dt + K_d\, \dot{r} \tag{44}$$

The output feedback using the PID controller is similar to any state feedback controller, as shown by Equation (41). In Eq. (41), ∫Ydt is introduced as a new state variable. The integral term has the following definition.

$$\mathrm{x} = \int_0^t \mathbf{y}\, dt \tag{45}$$

The variable then:

$$\dot{\mathrm{x}}_{\mathrm{new}} = \mathbf{y} = \mathbf{Cx} \tag{46}$$

The augmented state vector of the system is defined as

$$\bar{\mathrm{x}} = \begin{bmatrix}\mathbf{x} \\ \mathbf{x}_{\mathrm{new}}\end{bmatrix} \tag{47}$$

The augmented system may now be described as

$$\dot{\bar{\mathrm{X}}} = \bar{\mathrm{A}}\bar{\mathrm{x}} + \bar{\mathrm{B}}\mathbf{u} \tag{48}$$

The augmented system matrix $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ are described by

$$\bar{A} = \begin{bmatrix} \mathbf{A} & 0_{n\times m} \\ \mathbf{C} & 0_{m\times m} \end{bmatrix}, \bar{B} = \begin{bmatrix} \mathbf{B} \\ 0_{m\times r} \end{bmatrix} \quad (49)$$

The substituting Eq. (47) into the control input Eq. (41) is then

$$\mathbf{u} = \mathbf{u}_r - \mathbf{K}_{pid}\bar{x} \quad (50)$$

where $\mathbf{K}pid$ = [$\bar{\mathbf{K}}p\bar{\mathbf{K}}i$]. Substituting the Eq. (50) into (48) gives

$$\dot{\bar{x}} = (\bar{A} - \bar{B}\mathbf{K}_{pid})\bar{x} + \bar{B}\mathbf{u}_r \quad (51)$$

The eigenvalues of $\bar{A} - \bar{B}K_{PId}$ are placed in the left half of the complex s-plane for asymptotically stable. The performance index of the form given by Eq. (48)

$$J_{pid} = \int_{t_0}^{t_1} (\bar{x}^T\bar{Q}\bar{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})\, dt \quad (52)$$

The desired optimal control is:

$$\mathbf{u}^* = -\mathbf{R}^{-1}\bar{B}^T\bar{P}\bar{x} \quad (53)$$

where the matrix $\bar{\mathbf{P}}$ is defined as

$$\bar{P}\bar{A} + \bar{A}^T\bar{P} - \bar{P}\bar{B}\mathbf{R}^{-1}\bar{B}\bar{P} + \bar{Q} = 0 \quad (54)$$

where $\bar{\mathbf{Q}} \in \mathbb{R}^{(n+m)\times(n+m)}$ extended state weight matrix. Comparing the Eq. (41) and Eq. (53) concluded

$$[\bar{K}_p \quad \bar{K}_i] = \mathbf{R}^{-1}\bar{B}^T\bar{P} \quad (55)$$

where $\bar{\mathbf{K}}p \in \mathbb{R}^{(n+m)}$ and $\bar{\mathbf{K}}i \in \mathbb{R}^{(n+m)}$ Once Eq. (55) is obtained, the controller coefficient are all calculated as following expressions

$$[\mathbf{K}_p \quad \mathbf{K}_d] = \bar{K}_p\bar{C}^T(\bar{C}\bar{C}^T)^{-1} \quad (56)$$

Where

$$\bar{\mathbf{C}} = [C^T \ (CA - CB\bar{K}p)^T]^T \quad (57)$$

and

$$\mathbf{K}_i = (\mathbf{I}_m + \mathbf{K}_d\mathbf{C}\mathbf{B})\bar{K}_i \quad (58)$$

The last two equations conclude optimal PID controller parameters.

5. Results and discussions

In this section, the results of application of PID and LQR regulators on a Permanent Magnet DC Motor is described. When a torque of 0.003 Newton meters is applied to a permanent DC motor, the motor gradually accelerates until it reaches a steady speed. The steady-state speed of the motor depends on the motor's characteristics, such as its torque and speed constants, as well as the operating conditions, such as the load torque.

Several factors influence the speed of a permanent DC motor under applied torque, including:

- Applied Torque: The higher the applied torque on the motor, the faster its speed.
- Torque Constant (Tl): The torque constant defines the relationship between the current flowing through the motor and its generated torque. Km is associated with the motor's torque per ampere.
- Speed Constant (Kv): The speed constant establishes the relationship between the applied voltage to the motor and its resulting speed. Kv is associated with the motor's speed per volt.
- Armature Resistance (Ra): The armature resistance represents the opposition to the flow of electric current within the motor. A higher armature resistance leads to a decrease in motor speed.
- Armature Inductance (La): The armature inductance represents the opposition to changes in electric current within the motor. A higher armature inductance results in a slower response of the motor to voltage changes.

The physical constants of motor are shown in the following table:

Table 1 – Physical Constant of the Motor.

Parameter	Ra/	Va/	La/	Kv	Bm/	Tl/	j/

	Ω	v	H		N.m/rad/sec	N.m	KG.m²
Value	7	6	0.12	0.0141	$6.04*10^{-6}$	0.003	$1.61*10^{-6}$

5.1. Results with PID Control

For the synthesis of the PID regulator; the parameters P, I and D are adjusted and adapted using Ziegler-Nichols method:

Table 2 – PID regulators parameters.

Parameter	Kp	Ki	Kd
Initial Values	0.5	0.0001	0.001
Best Values	0.367836643736341	1.72811242839962	-0.0283263958477942

By applying a disturbance (load torque) to this system of the order of 0.003 [N.M] just when starting the engine to study its response at steady state.

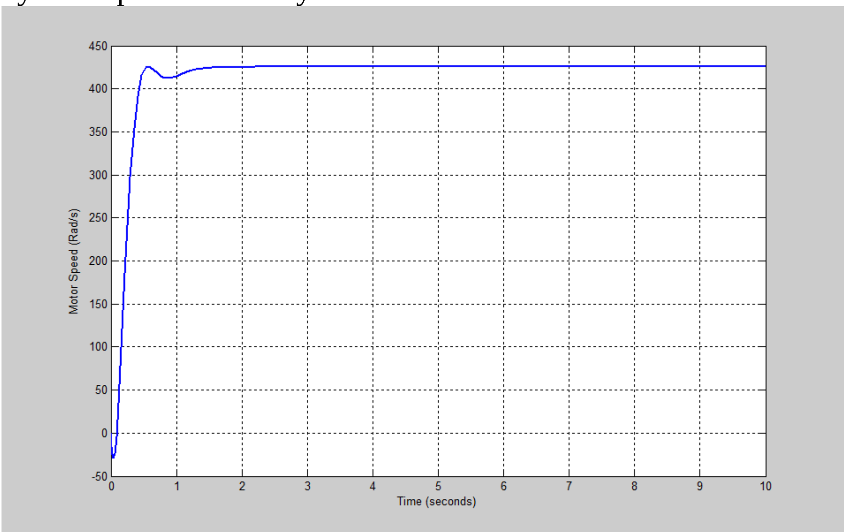

Figure 6 - DC Motor Speed Response with PID Regulator

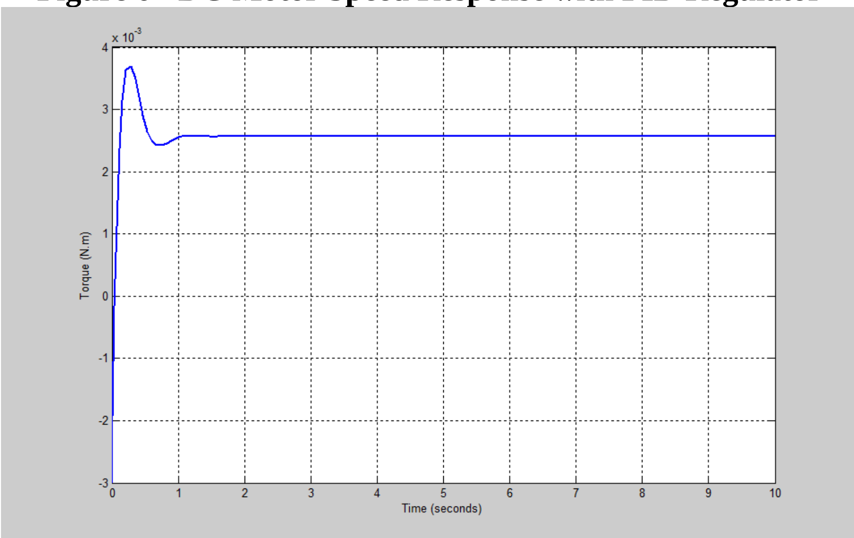

Figure 7 - Response of DC TORQUE using PID Regulator

Figure 6 shows the speed curve of a dc motor controlled using a PID controller. A PID (proportional-integral-derivative) controller is a common type of feedback controller widely used in various control systems, including DC motor control. It aims to achieve a desired system output by adjusting the control signal based on the error between the desired and actual output.

The speed curve depicts the motor's speed as a function of time. The speed curve converges towards the value of 150 [rad] at time 0.4 [s] and stabilizes there. This indicates the motor's speed, minimizing the error between the desired and actual speed.

The figure 7 shows the torque curve of the DC motor as function of time while using the PID controller. We observe that the motor torque starts from a value of 1.8 *10-3 [N.m], reses to value of 2.5*10-3 [N.m] at time 0 [s], then the torque decreases to a value of - 0.9*10-3 [N.m] at time 0.4 [s] and then stabilizes at the same value.

5.2. Results with LQR Controller

The simulation procedure may be summarized as follows:
First, input the DC motor data. Then, write the differential equations for the model then get the state space representation as is equation (21). After, get the open loop transfer function and the closed loop step response. Finally performing the performance of LQR controller and get the result. The results are presented in the following section:

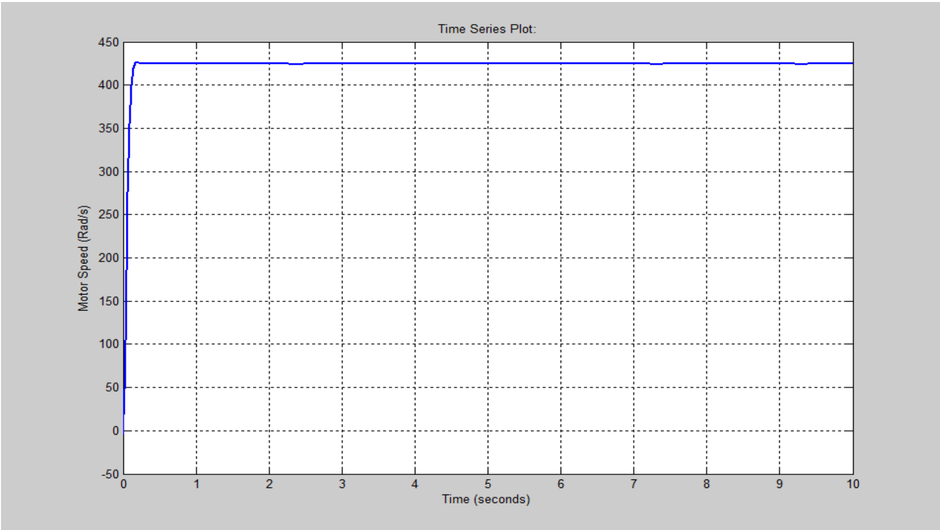

Figure 8 - DC Motor Speed Response with LQR Regulator

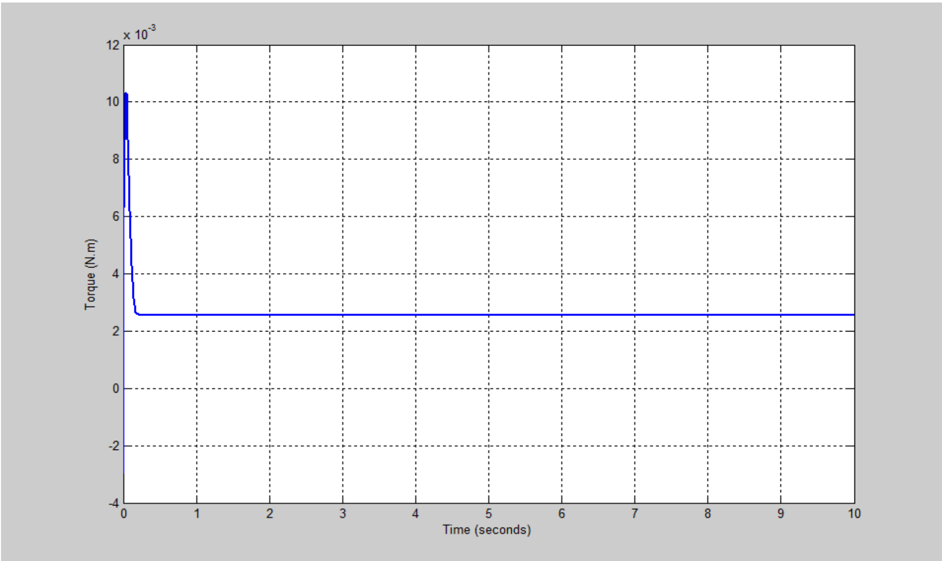

Figure 9 – DC Motor Torque Response with LQR Regulator

The speed of a DC motor controlled by a LQR controller is displayed in Figure 8. An example of an optimal controller that is employed to accomplish a certain system response is the linear quadratic regulator, or LQR, which can be used to regulate the speed and position precision of a DC motor.
The figure shows the motor speed as a function of time. The speed starts from a negative value of -30 [rad] at a time of 0 [s], then decreases to a value of -60 [rad] at a time of 0.2 [s], then increases

gradually until it reaches the value of 270 [rad]. Then the speed is maintained at the same. The curve also shows that the system response is fast and has little vibration.

Figure 9 shows the rotor torque response of a DC motor while using an LQR controller. We observe that the torque of the DC motor starts from a minimum value of -2.6 [N.m] at time 0 [s] and increases slowly to a value of 1.7 [N.m] at time approximately 0.2 [s] and then stabilizes at the same value.

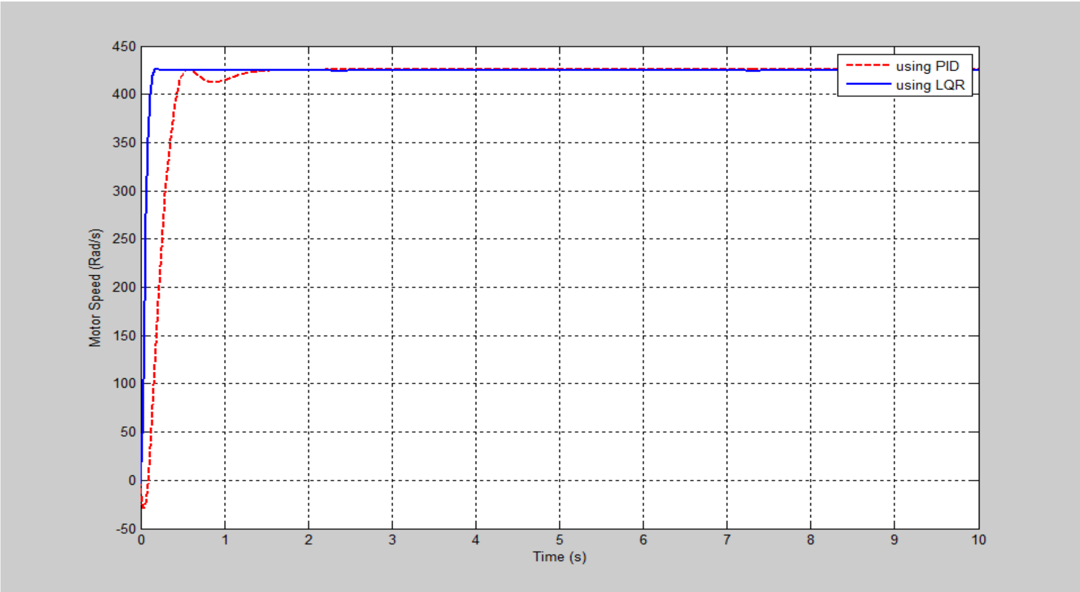

Figure 10 - DC Motor Speed Control with LQR and PID regulators

The results from the two strategies are briefly presented in table 3.

Table 3 – Performance comparison between both control strategies

Time Response specifications	LQR	PID
Settling Time (Ts) [s]	**17.8575**	**18.8575**
Rise Time (*Tr*)	**1.4061**	**1.4061**
Over shoot [%]	**15.8416**	**15.8416**
Under shoot [%]	**0**	**0**
Stability	**Stable**	**Stable**
Peak Time [s]	**12**	**13**

6. Conclusions

The purpose of this was to carry out an analysis and performance comparison to demonstrate that both LQR and PID controllers are effective in controlling the speed of a DC motor. However, LQR controllers offer faster response and higher accuracy, while PID controllers provide design simplicity, the choice of the appropriate controller depends on the specific system requirements.

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