

## Numerical analysis of shape coefficients and bearing capacity factors of shallow foundations on heterogeneous soil

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### Abstract

We often face heterogeneous foundation soils in geotechnics, which significantly influence shallow foundations' shape coefficients and bearing capacity factors. The evaluation of these factors introduces significant uncertainties in the thickness of the upper layer is comparable to the width of the rigid footing placed on the soil surface. This ambiguity depends on the geometry of the footing, the mechanical properties of the soil, and other geotechnical factors. Conducting thorough geotechnical studies and performing specific analyses are essential to accurately assess the effect of clay layering on the shape coefficients and bearing capacity factors of circular and square footings. A numerical analysis using the FLAC 2D and 3D software is carried out to evaluate the effect of the superposition of two undrained clay layers on the bearing capacity and shape factors of circular and square footings subjected to a static axial load. The aspects of homogeneous and heterogeneous shear strength profiles were addressed at various positions of the height of the first clay layer. It was found that the critical depth of the first clay layer necessary to optimize the bearing capacity of a foundation resting on stratified soil is 2, 1.5, and 1, respectively, for strip, square, and circular footings. The shape coefficient values  $s_c$  range between 1.14 and 1.22 for square footings and between 0.98 and 1.64 for circular footings. The results are compared to previously published values in the literature.

**Keywords:** Shape Coefficient, Bearing Capacity Factor, FLAC, Square Foundation, Circular Foundation, Failure.

### Resumé

En géotechnique on est souvent confrontés à des sols de fondation hétérogène qui influe significativement sur les coefficients de forme et les facteurs de portance des fondations peu profondes. L'évaluation de ces derniers, introduit des incertitudes significatives si l'épaisseur de la couche supérieure est comparable à la largeur de la semelle rigide placée à la surface du sol. Cette ambiguïté dépend de la géométrie de la semelle, des propriétés mécaniques du sol et d'autres facteurs géotechniques. Mener des études géotechniques approfondies et effectuer des analyses spécifiques sont essentiels pour évaluer avec précision l'effet de la stratification des argiles sur les coefficients de forme et les facteurs de portance d'une semelle circulaire et carré. Une analyse numérique utilisant le logiciel FLAC 2D et 3D est réalisée pour évaluer l'effet de la superposition de deux couches d'argile non drainée sur les facteurs de portance et de forme des semelles circulaires et carrés soumises à une charge axiale statique. Les aspects de profils de résistance au cisaillement homogènes et hétérogène ont été abordés à diverses positions de la hauteur de la première couche d'argile. Il est constaté que la profondeur critique de la première couche d'argiles nécessaire pour optimiser la capacité portante d'une fondation reposant sur un sol stratifié est de 2, 1.5 et 1 respectivement pour une semelle filante, carrée et circulaire. Les valeurs coefficient de forme  $s_c$  varient entre 1,14 et 1,22 dans le cas des semelles carrées et entre 0.98 et 1.64 dans le cas des semelles circulaires. Les résultats sont discutés et comparés aux valeurs précédemment publiées disponibles dans la littérature.

**Mots-clés:** Coefficient de Forme, Facteur de Portance, FLAC, Fondation Carrée, Fondation Circulaire, Rupture.

### 1. Introduction

The bearing capacity analysis is based on Terzaghi's superposition theory under plane strain conditions. Experimental and theoretical studies show that the bearing capacity of shallow foundations increases when three-dimensional (3D) effects are considered. So far, only the axisymmetric case of circular foundations has been theoretically solved, showing that, in the case of purely cohesive soils, the bearing capacity increases by a factor of 1.2 compared to the two-dimensional (2D) case (Vesic A. S. 1975). On the other hand, the solutions mainly rely on applying empirical formulas for shape coefficients to consider other geometric shapes of foundations. Numerical calculations using Flac2D and Flac3D code are conducted to evaluate the bearing capacity factors  $N_c$  and shape coefficients  $s_c$  of square and circular footings under static loading. These factors are related to the relative thickness of the upper layer and the undrained cohesion ratio of clayey soils arranged in two layers. The shape coefficient  $s_c$  represents the ratio between the three-dimensional bearing capacity factor  $N_c^*$  of the square or circular footing and the two-dimensional bearing capacity factor of a strip footing  $N_c$ . Based on the plane strain assumption, the bearing capacity calculation theory is established for a strip footing resting on homogeneous soil. However, the soils in contact with the foundation base are generally heterogeneous and characterized by various natures and behaviors. Authors who have addressed this issue, making assumptions about the roughness of the footing and the shape of failure mechanisms, all agree on the general formula proposed by Terzaghi K. (1943).

$$q_u = cN_c + qN_q + \frac{1}{2} \gamma B N_\gamma \quad (1)$$

$N_c$ ,  $N_q$ , and  $N_\gamma$  are the bearing capacity factors.

For other geometric shapes of foundations, the bearing capacity is determined by introducing empirical shape coefficients  $s_c$ ,  $s_q$ , and  $s_\gamma$  to account for the three-dimensional effect.

$$q_u = cs_c N_c + qs_q N_q + \frac{1}{2} \gamma Bs_\gamma N_\gamma \quad (2)$$

$s_c$ ,  $s_q$ , and  $s_\gamma$  are the shape coefficients.

If the foundation rests directly on the free surface of the cohesive undrained clay soil  $c_u$ , equation (2) becomes a function solely of the undrained cohesion and is independent of the soil unit weight.

$$q_u = c_u s_c N_c \quad (3)$$

The calculation of the ultimate load naturally depends on the determination of  $N_c$ , which is based on various assumptions primarily related to the chosen failure mechanism configuration.

Different authors have focused on calculating the ultimate load by considering the shape of the foundation and have proposed empirical relationships for the correction factor  $s_c$ . A general overview of the literature reveals notable divergences.

Terzaghi, (1943) proposed an empirical formula for the shape coefficient  $s_c$  in the case of a rectangular or square footing :  $s_c = 1 + 0.2 \frac{B}{L}$ . He suggests a value of 1.2 for a square footing and 1.3 for a circular footing. These values were likely derived from the tests by Golder (1941) and additional unpublished data. The tests were conducted on square footings resting on clay.  $s_c$  is calculated as the ratio of the ultimate load of a square footing with dimensions 760x760 mm<sup>2</sup> to that of a rectangular footing with dimensions 4570x760 mm<sup>2</sup>.

Experiments conducted primarily by Meyerhof (1951, 1963) and de Beer (1970) on scale models of circular and square footings resting on undrained clay led to the proposal of an empirical formula for the shape coefficient  $s_c$ .

$$s_c = 1 + \frac{B}{L} \frac{N_q}{N_c} \quad (4)$$

In this case, where  $\varphi = 0$ , the bearing capacity factors are  $N_q = 1$  and  $N_c = 2 + \pi$ . Thus, the shape coefficient  $s_c = 1.194$ . Brinch Hansen (1970) proposes the value of  $s_c = 0.8$  and Lancellotta (1995) suggests the value of  $s_c = 1.2$ . It should be noted that most European codes (2000) recommend taking  $s_c = 1.20$  for a circular or square footing, thus aligning with the formula proposed by Terzaghi.

Salgado et al. (2004) studied the bearing capacity of differently shaped foundations placed on clay using finite element limit analysis. The results of these analyzes led to an empirical formula for the shape coefficient  $s_c$ , they proposed that for :  $1 \leq \frac{B}{L} \leq 5$  and  $0 \leq \frac{D_f}{B} \leq 1$

$$s_c = 1 + 0.12 \frac{B}{L} + 0.17 \sqrt{\frac{D_f}{B}} \quad (5)$$

Where  $D_f$  is the depth of footing embedment.

Zhu and Michalowski (2005) estimated  $s_c = 1.06$  for square footings using the FLAC3D code. Using the finite element code ABACUS, R.S. Merifield and V. Q. Nguyen (2006) obtained  $s_c = 1.17$ .

Meyerhof and Hansen acknowledge the influence of the internal friction angle  $\varphi$  on the shape factors. In contrast, Terzaghi, Vesic, and Lancellotta ignore it and only relate these factors to the foundation geometry. Ghiasi V. and Sohrabi F. (2022) and Garcez Fonseca, J.F. and Fernandes Azevedo, G. (2024) examined the optimization of bearing capacity and settlement of shallow foundations. They compared deterministic and probabilistic methods, considering variables such as foundation dimensions (L/B) and the soil depth ratio under the foundation (H/B), in soils with different elastic properties and loading conditions. Theoretically, all these approaches will estimate an upper limit of the shape factor coefficient, as they all rely on failure mechanisms to calculate  $N_c$ , except for the case of (Brinch Hansen 1970), which likely constitutes a lower limit.

In general, the ultimate bearing capacity of square and circular footings resting on a single layer of undrained homogeneous clay can be estimated using equation (3) with equation (4) or (5). However, the soil beneath the foundation is not homogeneous; it consists of distinct layers with significantly different properties. The effect of soil stratification on continuous footings has been studied by several researchers, including Button (1953), Reddy and Srinivasan (1967), Brown and Meyerhof (1969), Chen (1975), Meyerhof and Hanna (1978), and Merifield et al. (1999). However, no rigorous solutions appear for the problem of circular and square footings resting on stratified clays. Geotechnical engineers have addressed this issue by simply averaging the resistances of the different layers or by adopting high safety factors to account for the uncertainty related to soil stratification.

For a stratified soil profile, it is useful to rewrite equation (3) as follows :

$$q_u = c_{ul} s_c N_c^* \quad (6)$$

Where  $c_{u1}$  is the undrained shear strength of the upper layer and  $N_c^*$  is the modified bearing capacity factor that depends on both the depth  $H$  of the first layer and the strength ratio of the two superimposed layers  $\frac{c_{u1}}{c_{u2}}$ . The values of  $N_c^*$  are determined using simulations performed with FLAC for each ratio of  $\frac{H}{B}$  and  $\frac{c_{u1}}{c_{u2}}$ . The shape coefficient  $s_c$  is derived from the following relationship :

$$s_c = \frac{N_c^*}{N_c} \quad (7)$$

## 2. Numerical simulations

The discretization of the bearing capacity problem in this study is illustrated in Figure 1 (a) and (b); it represents the strip footing of width  $B$ , the circular footing of diameter  $D$ , and the square footing of side  $B$ . Each one rests on the free surface of a layer of undrained cohesive clay  $c_{u1}$  with a thickness  $H$ , overlying a second layer of undrained cohesive clay  $c_{u2}$  with a thickness large enough to neglect the influence of the edges. Due to the symmetry of the strip footing, the axisymmetry of the circular footing, and the double symmetry of the square footing, only half and a quarter of the domain are discretized separately. A continuous mesh of the domain is chosen due to the perfect roughness at the interface of the two clay layers. Due to the concentration of shear stresses near the edges of the footing, the mesh is refined in this region. The extent of the adopted boundaries is sufficient to ensure that the failure mechanism does not intersect them and does not influence the results. The lower boundary has been considered fixed, while the vertical boundaries have been restricted only in movement in the horizontal direction. A Mohr-Coulomb-type plasticity criterion is adopted with a non-associated flow rule. The calculation of  $N_c^*$  depends on two dimensionless parameters, namely the ratios of  $\frac{H}{B}$  et  $\frac{c_{u1}}{c_{u2}}$ , varying respectively from 0.2 to 2 and 0.25 to 5. Such results have already been discussed in the literature (S. Benmebarek *et al.*, 2012 and S. Benmoussa *et al.*, 2018), and Nezzari *et al.*, 2024), this covers most practical problems of interest. Since the footing is considered rigid, the simulation method involves applying a very slow displacement velocity to the nodes of the footing in contact with the ground until a state of plastic flow is reached. This approach allows the model to adapt to this disturbance and avoids numerical instabilities. The equivalent pressure is recorded during the footing displacement, and the ultimate load is determined when the soil reaches a plastic flow consistent with the chosen criterion. The soil/foundation interface is simulated as perfectly rough, thus blocking the horizontal displacement of the nodes of the footing in contact with the soil.

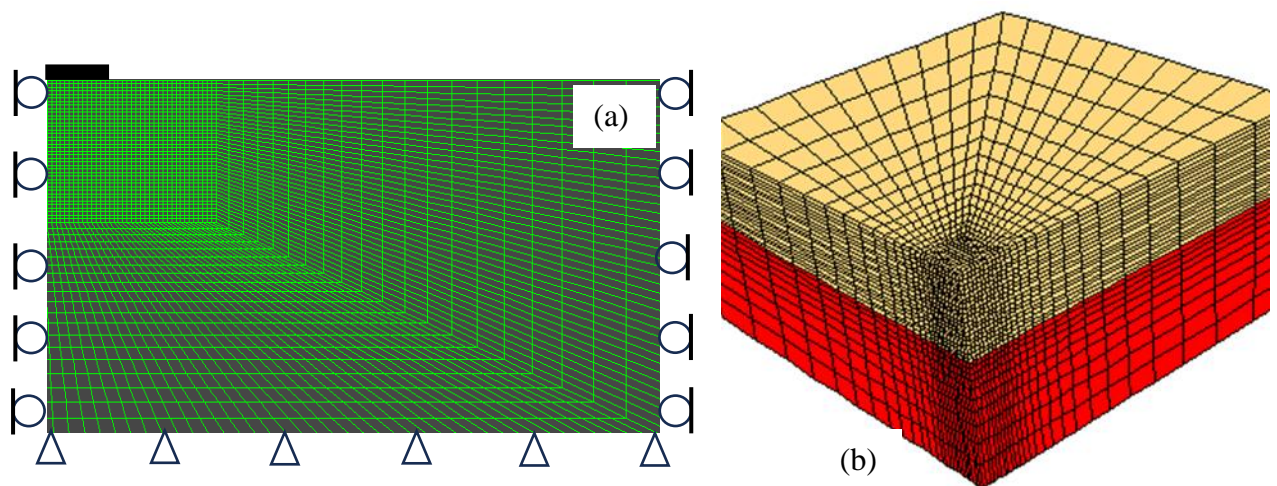


Figure 1 - The mesh used

(a) continuous or circular footing  
(b) square footing

### 3. Results and comments

While the results obtained using the FLAC3D calculation code are logical to some extent, they nevertheless differ from those proposed in the literature. Figures 2 to 6 respectively represent the calculated bearing factors  $N_c$ ,  $N_c^{cir}$  and  $N_c^{sq}$  for strip, circular, and square footings. They highlight the influence of the ratios of strength between the two layers of clay  $\frac{c_{u1}}{c_{u2}}$ , the depth of the first layer  $H$ , and the geometric shape of the footing. The values of the bearing factor and shape coefficients for each type of foundation are listed in Table 1., for homogeneous clay soil for comparison purposes, and in Table 2., for heterogeneous soil. These data are also graphically represented to facilitate their use. To validate our simulation procedure, the case of a strip footing resting on a homogeneous clay profile was studied and compared to the exact solution of Prandtl ( $2 + \pi$ ). The value of  $N_c$  at the steady state yields a value of 5.16, with a relative error of 0.4%.

**Table 1 – Comparison of  $N_c$  values with those obtained by other authors for the case of homogeneous clay soil.**

	Current study		Merifield et al (2006)		Salgado et al (2004)			
	Square	Circular	Square	Circular	Square		Circular	
					Lower bound	Upper bound	Lower bound	Upper bound
$N_c^*$	5.95	6.05	5.95	6.05	5.523	6.221	5.856	6.227
$s_c$	1.15	1.17	1.194		1.12		1.12	

For an initial verification of the three-dimensional model, the bearing factor of square and circular footings resting on the surface of homogeneous soil was calculated and compared with existing published numerical results. The bearing factors for the square footing ( $N_c^{sq} = 5.95$ ) and for the circular footing ( $N_c^{cir} = 6.05$ ) are the same as those reported by R. S. Merifield et al. (2006). They compare well with those calculated using the widely adopted shape factor in equation (4) and the solutions of Salgado et al. (2004). The bearing capacity factor for square footings,  $N_c^{sq} = 5.95$ , was approximately 2% lower than that of circular footings,  $N_c^{cir} = 6.05$ . This observation is consistent with the results of Merifield et al. (2006) and Salgado et al. (2004) as indicated in Table 1. The bearing capacity factor for strip footings was 5.16, approximately 0.4% higher than the classical solution of Prandtl of ( $2 + \pi$ ). The calculated shape factor,  $s_{cs\_csc}$ , using equation (7), is therefore 1.17 and 1.15 respectively for circular and square footings on a homogeneous clay profile.

**Table 1 – Values of bearing capacity factors  $N_c$  and shape coefficients  $s_c$ .**

H/B	$c_{u1}/c_{u2}$	present study					Merifield et al (2006)	
		Streep	Square	Circular	$s_c^{sq}$	$s_c^{cir}$	Square	Circular
0.2	0.25	6.50	7.41	6.39	1.14	0.98	6.35	6.36
	0.5	6.50	7.41	6.39	1.14	0.98	6.35	6.36
	0.75	6.31	7.21	6.38	1.14	1.00	6.27	6.34
	1	5.16	5.95	6.05	1.15	1.17	5.95	6.05
	1.25	4.39	5.09	5.59	1.16	1.27	5.45	5.59
	1.5	3.78	4.41	5.14	1.16	1.36	5.03	5.17
	2	3.08	3.63	4.45	1.18	1.45	4.39	4.51
	2.5	2.61	3.10	3.90	1.19	1.49	3.92	4.02
	3	2.28	2.73	3.50	1.20	1.54		
	4	1.85	2.22	2.93	1.20	1.58	3.04	3.13
0.5	5	1.75	2.10	2.59	1.20	1.48	2.70	2.78
	0.25	5.43	6.19	6.05	1.14	1.11	5.96	6.04
	0.5	5.43	6.19	6.05	1.14	1.11	5.96	6.04
	0.75	5.43	6.19	6.05	1.14	1.11	5.96	6.04
	1	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	1.25	4.70	5.47	6.03	1.16	1.28	5.94	6.02

	1.5	4.29	5.05	5.91	1.18	1.38	5.82	5.90
	2	3.69	4.41	5.38	1.20	1.46	5.46	5.58
	2.5	3.28	3.93	4.97	1.20	1.52	5.08	5.23
	3	2.97	3.58	4.62	1.21	1.56		
	4	2.53	3.06	4.10	1.21	1.62	4.22	4.39
	5	2.23	2.71	3.82	1.22	1.71	3.89	4.03
1	0.25	5.16	5.95	6.05	1.15	1.17	5.93	6.03
	0.5	5.16	5.95	6.05	1.15	1.17	5.93	6.03
	0.75	5.16	5.95	6.05	1.15	1.17	5.93	6.03
	1	5.16	5.95	6.05	1.15	1.17	5.95	6.05
	1.25	5.16	5.94	6.05	1.15	1.17	5.94	6.05
	1.5	4.98	5.71	6.04	1.15	1.21	5.94	6.05
	2	4.44	5.11	6.04	1.15	1.36	5.93	6.06
	2.5	4.17	4.80	6.03	1.15	1.45	5.93	6.06
	3	3.98	4.58	6.00	1.15	1.51		
	4	3.59	4.07	5.90	1.13	1.64	5.86	6.04
	5	3.29	3.72	5.83	1.13	1.77	5.77	5.94
1.5	0.25	5.16	5.95	6.05	1.15	1.17	5.94	6.04
	0.5	5.16	5.95	6.05	1.15	1.17	5.94	6.04
	0.75	5.16	5.95	6.05	1.15	1.17	5.94	6.03
	1	5.16	5.95	6.05	1.15	1.17	5.95	6.05
	1.25	5.16	5.95	6.05	1.15	1.17	5.94	6.04
	1.5	5.16	5.93	6.05	1.15	1.17	5.94	6.04
	2	5.16	5.83	6.05	1.13	1.17	5.94	6.04
	2.5	5.05	5.61	6.05	1.11	1.20	5.94	6.04
	3	4.85	5.36	6.05	1.11	1.25		
	4	4.53	4.94	6.05	1.09	1.34	5.94	6.04
	5	4.25	4.61	6.05	1.08	1.42	5.94	6.03
2	0.25	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	0.5	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	0.75	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	1	5.16	5.95	6.05	1.15	1.17	5.95	6.05
	1.25	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	1.5	5.16	5.95	6.05	1.15	1.17	5.96	6.05
	2	5.16	5.93	6.05	1.15	1.17	5.96	6.05
	2.5	5.16	5.91	6.05	1.15	1.17	5.96	6.05
	3	5.16	5.85	6.05	1.13	1.17		
	4	5.08	5.64	6.05	1.11	1.19	5.96	6.05
	5	5.01	5.40	6.05	1.01	1.21	5.96	6.05

At first glance, the bearing capacity factor values calculated by FLAC for circular footings compare well with those found by Merifield et al. (2006) ; they are almost identical. For square footings, the values are slightly lower, with a variation rate of around 15%.

### 3.1 Footing on the free surface of a layer of stiff clay overlying another soft one

When the upper layer is stronger than the lower layer, an increase in the bearing capacity factor  $N_c$  occurs simultaneously with a decrease in the strength ratio  $\frac{c_{u1}}{c_{u2}}$  and an increase in the ratio  $\frac{H}{B}$ . It converge respectively to 5.16, 5.95, and 6.05 for strip, square, and circular footings. A good agreement is observed between the proposed values and those recommended by Mir, M. and Bouafia, A. (2024), who studied the cone factor  $N_k$  from the cone penetration test (CPT) as well as the vertical response of shallow foundations in saturated clays. These values were used to derive the bearing capacity factor  $K_c$  for a shallow foundation. A critical depth is observed beyond which the bearing capacity factor  $N_c$  becomes constant :  $\frac{H}{D}=1.5$  for a circular footing and  $\frac{H}{B}=2$  for square and strip footings. For strong clay overlying soft clay profile, the larger the ratio  $\frac{c_{u1}}{c_{u2}}$  is, the larger the

critical depth will be. This critical depth  $\frac{H}{D} = 1$  for circular footing is significantly less than the  $\frac{H}{B} = 2 \approx 2.5$  for square and strip footing found by some investigators (Salgado, R. et al. 2004, Benmebarek, S. et al. 2012, Benmoussa, S. et al. 2012). In this case, the failure mechanism occurs primarily in the upper layer, and the entire soil is considered homogeneous, taking into account only the properties of the upper layer (Figure 7). Under conditions where  $\frac{c_{u1}}{c_{u2}}$  and  $\frac{H}{B} = 0.2$ , the minimum value of  $N_c$  is 2.01, 1.57, and 2.59 for square, strip, and circular footings respectively. For the same strength parameters at a depth of  $\frac{H}{B} = 0.5$ , the minimum value of  $N_c$  is 2.71, 2.23, and 3.82 for square, strip, and circular footings respectively. When the upper layer gains significant strength compared to the lower layer, a complete shear failure through the upper layer commonly occurs, especially for ratios  $\frac{H}{B} \leq 0.5$  (Figure 8). The shape coefficient depends closely on the  $\frac{H}{B}$  ratio. For a homogeneous soil, it generally stands around 1.15 for a square footing and 1.17 for a circular one. However, with an  $\frac{H}{B}$  ratio  $\leq 2$ , this coefficient can increase significantly, reaching about 1.2 for a square footing and 1.48 for a circular footing, especially if the upper layer becomes significantly stronger than the lower layer. These values seem to be well-defined compared to those referenced in the literature. The maximum values of  $s_c$  are around 1.22 and 1.77 when the ratio  $\frac{c_{u1}}{c_{u2}}$  is 5, for  $\frac{H}{B}$  ratios between 1 and 0.5, respectively, for a square and circular footing.

### 3.2 Footings at the free surface of a soft clay layer overlying a stronger layer.

When the upper layer of clay is weaker than the lower layer, the bearing factor  $N_c$  decreases as the ratio  $\frac{c_{u1}}{c_{u2}}$  and the  $\frac{H}{B}$  ratio simultaneously increase. However, as soon as the  $\frac{H}{B}$  ratio reaches 0.5, in this case,  $N_c$  reaches constant values of 6.19, 5.43, and 6.05 respectively for square, strip, and circular footings. At a  $\frac{H}{B}$  ratio of 0.2 and when  $\frac{c_{u1}}{c_{u2}}$  changes from 0.25 to 0.75,  $N_c$  decreases from 7.41, 6.39, and 6.5 to 7.21, 6.38, and 6.31 respectively for square, circular, and strip footings. For thin upper layers ( $H/B \leq 0.2$ ), the increase in the bearing factor  $N_c^*$  compared to the homogeneous case is significant, as illustrated in Figure 2. For thin upper layers where  $\frac{H}{B} \leq 0.2$  and  $\frac{c_{u1}}{c_{u2}} = 0.25$ , the failure mechanism slightly affects the lower layer, as shown by the concentration of shear stress increments and displacement vectors, as discussed by Danish *et al.* (2022) and illustrated in Figure 9. In cases where the ratio  $H/B > 0.5$ , however, the results indicate no increase in the bearing capacity of the three foundations. The critical depth, in this instance, seems to be a constant value around  $0.5B$ , smaller than in the scenario of a strong clay profile overlying a soft clay profile. The failure mechanism remains entirely confined to the upper layer, as confirmed by the concentration of shear stress increments and displacement vectors visualized in Figure 7. Therefore, the entire soil can be treated as a homogeneous medium, considering only the properties of the upper soil layer. In other words, as the thin upper layer becomes increasingly softer compared to the underlying solid layer, the contribution of the underlying layer to the ultimate bearing capacity increases. For a square footing, the shape coefficient  $s_c$  is constant and equal to 1.14. In contrast, a circular footing varies from 0.98 to 1.11 as the  $\frac{H}{B}$  ratio increases from 0.2 to 0.

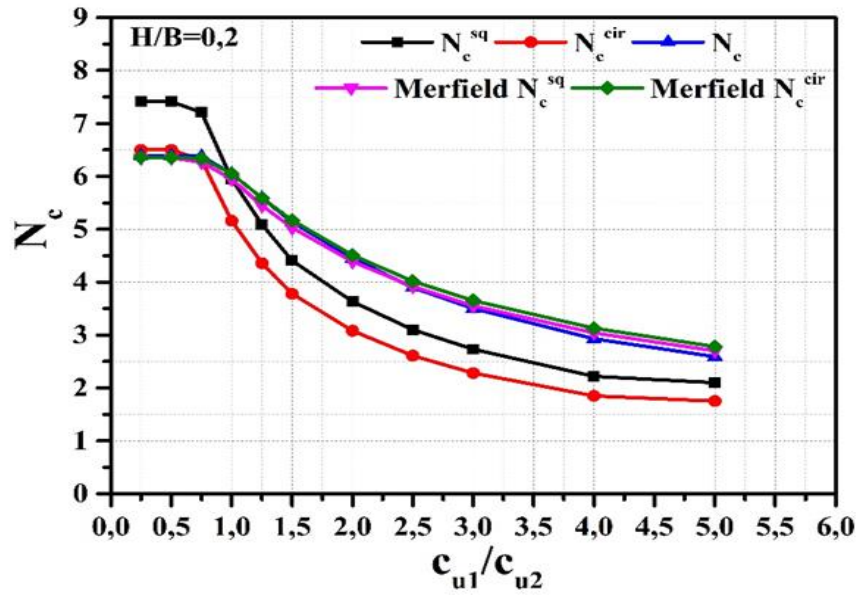


Figure 2. Variation of  $N_c$  as a function of the cohesion ratio for  $H/B = 0.2$

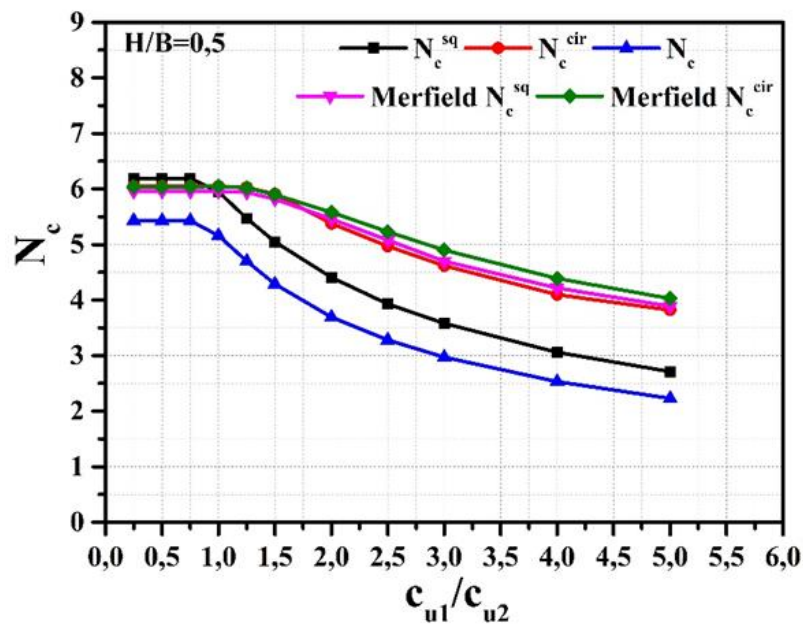


Figure 3. Variation of  $N_c$  as a function of the cohesion ratio for  $H/B = 0.5$ .



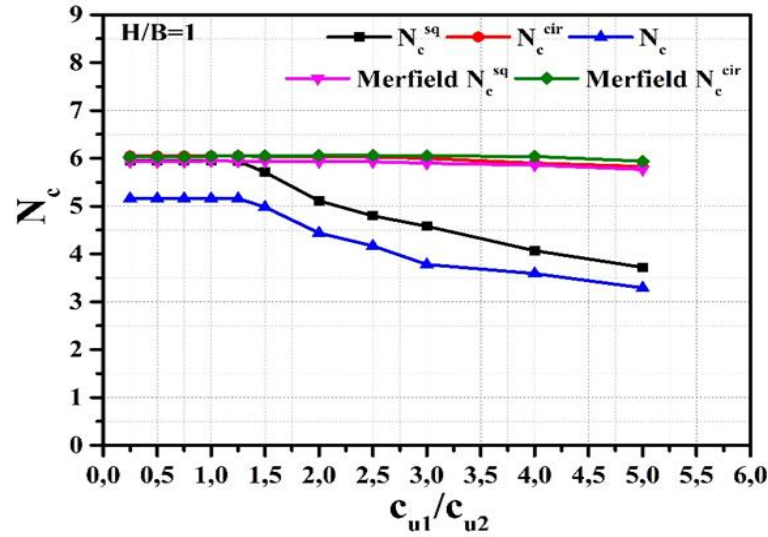


Figure 4. Variation of  $N_c$  as a function of the cohesion ratio for  $H/B = 1$

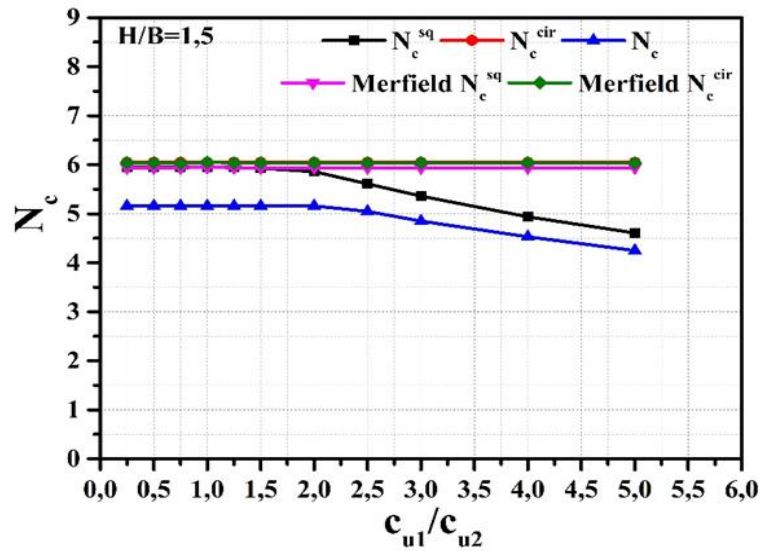


Figure 5. Variation of  $N_c$  as a function of the cohesion ratio for  $H/B = 1.5$

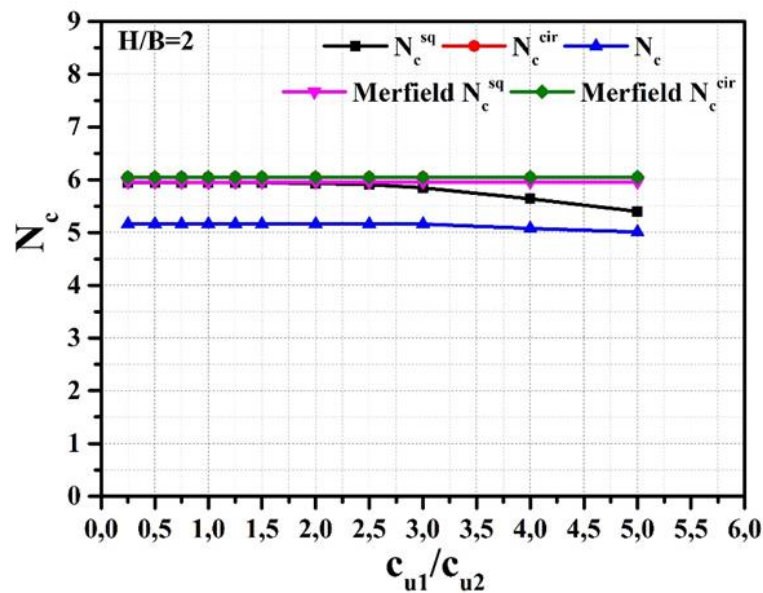
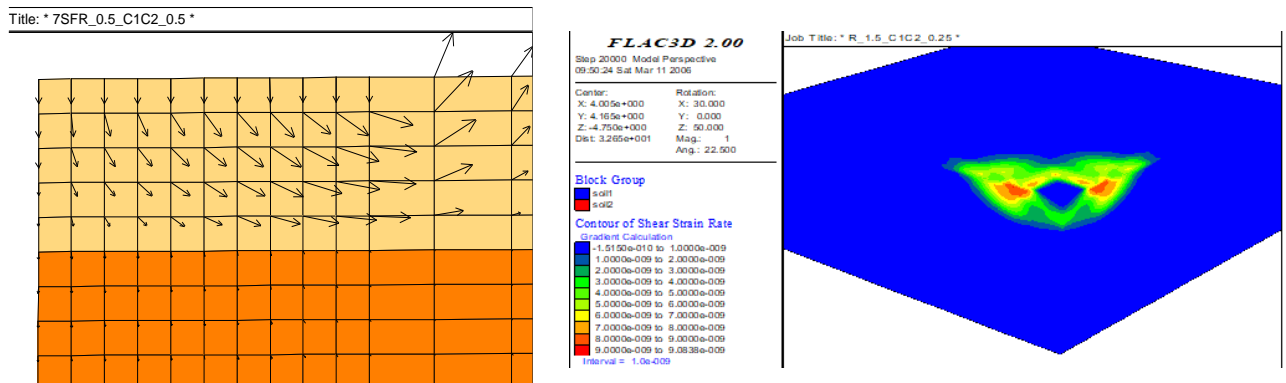
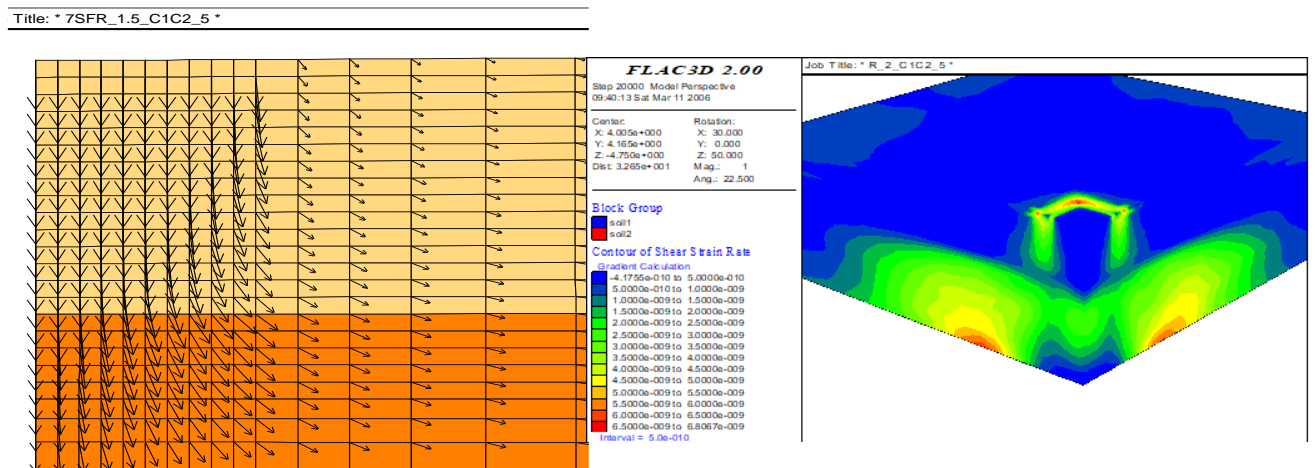


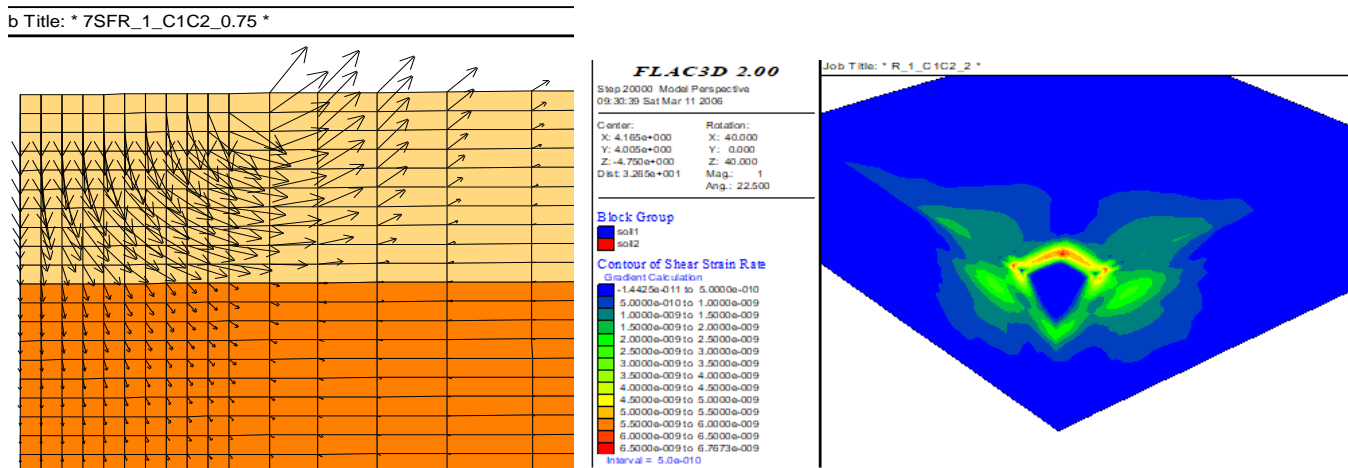
Figure 6. Variation of  $N_c$  as a function of the cohesion ratio for  $H/B = 2$



**Figure 7.** Failure mechanism visualized by the distribution of maximum shear strain rates and displacement field vectors with  $H/B = 0.25$  and  $c_{u1}/c_{u2} = 0.25$



**Figure 8.** Failure mechanism visualized by the distribution of maximum shear strain rates and displacement field vectors with  $H/B = 0.25$  and  $c_{u1}/c_{u2} = 0.5$



**Figure 9.** Failure mechanism visualized by the distribution of maximum shear strain rates and displacement field vectors with  $H/D = 0.5$  and  $c_{u1}/c_{u2} < 1$

#### 4. Conclusion

Several conclusions can be drawn from this investigation :

- ✓ The bearing capacity factors and shape coefficients mainly depend on the strength ratio of clay soils and the critical depth of the upper layer.
- ✓ The effect of a two-layered clay soil on the value of shape factors is well demonstrated.
- ✓ In the case of a homogeneous soil,  $s_c$  remains constant and is equal to 1.15 for a square footing and 1.17 for a circular footing.
- ✓ In the case of a layer of soft clay overlying a stronger one,  $s_c$  increases with the increase in the  $\frac{H}{B}$  ratio without exceeding the value obtained in the case of homogeneous soil.
- ✓ In contrast, in the case of a layer of strong clay overlying a soft one,  $s_{cs\_csc}$  increases progressively with the simultaneous increase of the ratios  $\frac{c_{u1}}{c_{u2}}$  and  $\frac{H}{B}$ , reaching a maximum value of approximately 1.22 for a square footing and 1.71 for a circular footing.
- ✓ The current numerical calculations of  $N_c^*$  for single-layer clay profiles show favorable agreement with established analytical and finite element solutions.
- ✓ In the context of a strong clay over soft clay profile, the results reveal a critical depth ratio of  $H/B=2$  to  $2.5$  for square and strip footings, notably higher than the  $H/B=1.5$  threshold observed for circular footings.
- ✓ The critical depth ratio in soft clay over a strong clay profile is about  $H/B=0.5$ , significantly lower than that observed when a strong clay layer overlies a soft clay profile.
- ✓ Different failure mechanisms have been observed, depending on the ratios  $c_{u1}/c_{u2}$  and  $H/B$  ; they are localized in the first layer or extend through it into the second layer. However, at a critical depth, the entire failure mechanism is localized in the upper layer.

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