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Influence of degree of saturation on the thermal conductivity of soils: Experimental and comparative model analysis

Influence du degré de saturation sur la conductivité thermique des sols: analyse expérimentale et comparative des modèles

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Abstract
The thermal conductivity λ can characterize thermal behaviour of soils in different engineering studies: environmental, geothermal, geotechnical and buildings construction. However, accurately measuring and predicting this parameter poses a challenging task. Measuring this parameter can be very complex, considering several factors, such as soil heterogeneity and external climatic conditions. In addition, predictions models may not capture all the nuances of real-world soil conditions, leading to less accurate predictions of λ. The objectives of this paper encompassed two primary aspects: (*i*) Evaluation of laboratory tests of thermal conductivity of unsaturated soil. (*ii*) Assessment of five highly recommended soil thermal conductivity models to determine their validity and strengthen their trustworthiness. The studied material consisted of calcareous tufa locally available in Beni-Saf region (Algeria). The tested samples were compacted to the Standard and Modified Proctor Optimum (SPO, MPO) followed by drying periods under laboratory conditions. The thermal conductivity of samples was evaluated using transient method. The models' predictive results of the thermal conductivity were assessed with different criteria such as Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Coefficient of Determination R^2.

Furthermore, this work also attempts to deliver an in-depth discussion of the effect of degree of saturation S_r on the thermal conductivity of soils.
Keywords: Unsaturated soils. Calcareous tufa. Thermal conductivity. Degree of saturation. Laboratory tests. Model predictions.

Résumé
La conductivité thermique λ peut caractériser le comportement thermique des sols dans divers disciplines de l'ingénierie : environnemental, géothermique, géotechnique et construction de bâtiments. Cependant, mesurer et prédire ce paramètre avec précision posent un défi majeur. La mesure de ce paramètre peut être très complexe, en tenant compte de plusieurs facteurs, tels que l'hétérogénéité du sol et les conditions climatiques externes. Les modèles peuvent ne pas saisir toutes les subtilités des conditions réelles des sols, ce qui conduit à des prévisions moins précises de λ. Les objectifs de ce travail comprennent deux aspects principaux : (i) Évaluation des résultats expérimentaux de la conductivité thermique des sols non saturés. (ii) Évaluation de cinq modèles recommandés de conductivité thermique des sols, documentés dans la littérature, afin de déterminer leur validité et de renforcer leur fiabilité. Le matériau étudié était du tuf calcaire disponible localement dans la région de Beni-Saf (Algérie). Les échantillons testés ont été compactés selon les Optimums de Proctor Normal et Modifié (OPN, OPM), suivis par des périodes de séchage en conditions de laboratoire. La conductivité thermique des échantillons a été évaluée en utilisant la méthode transitoire. Les résultats des modèles de la conductivité thermique ont été évalués avec différents critères tels que l'Erreur Absolue Moyenne (EAM), l'Erreur Quadratique Moyenne (EQM) et le Coefficient de Détermination R^2. Ce travail tente également de fournir une discussion approfondie de l'effet du degré de saturation S_r sur la conductivité thermique des sols.
Mots-clés : Sols non saturés. Tuf calcaire. Conductivité thermique. Degré de saturation. Tests en laboratoire. Prédictions de modèles.

1. Introduction

Natural soils serve as fundamental materials in environmental science, earth science, and various engineering applications. This can include designing of geothermal structures, such as Ground Source Heat Pumps (GSHPs) and Borehole Thermal Energy Storage (BTES) (Zhang and Wang 2017). In addition, soils can be employed as sustainable buildings materials using earth-based techniques such as: Rammed Earth (RE) or Compressed Earth Brick (CEB) (Delgado and Guerrero 2007; Schroeder 2016). A fundamental characteristic that describes heat transfer in soils is the thermal conductivity λ (Bristow 2002; He *et al*. 2017). The thermal conductivity defines the amount of heat, resulting from a temperature gradient, transmitted across a unit of thickness in a perpendicular direction of unit area (Chen *et al*. 2020). Thermal conductivity of materials can cover an extensive range (10^{-6} to 1000 [$W.m^{-1}.K^{-1}$]). However, monitoring this parameter in-situ or in laboratory as a function of the hydric state can be very challenging. Thus, several studies have made much effort to develop models based on easily measurable soil properties such degree of saturation S_r (Kersten 1949; Johansen 1977; Côté and Konrad 2005). The thermal conductivity models distinguish into two groups: Theoretical models and Empirical models (Xiong *et al*. 2023). Theoretical models are formulated by calculating λ for each component along with their corresponding volume fractions (Jia *et al*. 2019). One of the earliest theoretical models was that of Maxwell (Maxwell *et al*. 1904). Later, De Vries (1963) developed a model analogous to Maxwell. De Vries model offers a theoretical description of thermal conductivity as the weighted average of thermal conductivities of each soil component, and numerous studies have investigated this model (Farouki 1982; Haigh 2012; Tong *et al*. 2016). However, choosing proper parameters, such as shape factors, is required for accurate prediction of λ (Ochsner *et al*. 2001). However, empirical models try to establish correlations of λ through relationships between thermal conductivities of dry and saturated states of soil. The existing literature documents various empirical models, categorizing them into: (*i*) curve-fitting models and (*ii*) normalized thermal conductivity models. The curve-fitting models typically involves linear or non-linear regression of measured datasets, such as the works of (Kersten 1949; Alrtimi *et al*. 2016). While their applications are simple, the lack of flexibility often restricts their applications to specific soil types (He *et al*. 2020). Kersten (1949) established an empirical model based on an extensive number of laboratory measurements. His study evaluated the thermal conductivity of 19 soils with different densities, moisture content, and temperatures. Lu and Ren (2009) highlighted that Kersten model was not suitable to predict λ at lower water contents. Alternatively, the normalized thermal conductivity λ model proposed by Johansen (1977) demonstrated greater flexibility and has found extensive application (Farouki 1981; Côté and Konrad 2005; Hu *et al*. 2001; Lu *et al*. 2007; Chen *et al*. 2020). In Johansen's study, a simple empirical model was developed based on soil mineral composition and degree of saturation S_r. For many soils, Johansen's model offered accurate predictions of thermal conductivity (Farouki 1981; Farouki 1982; Zhang 2017). The work of Côté and Konrad (2005) developed a generalized model, based on Johansen work, for estimating thermal conductivity of natural soils. Later, Lu and Ren (2009) assessed the Côté and Konrad model and highlighted its lower performance on fine-textured soils. The study of Barry-Macaulay *et al*. (2015) compared the original Johansen model to its three derivative models established by Côté and Konrad (2005), Balland and Arp (2005), Lu and Ren (2009). Results indicated that all three models presented well-comparable fits to experimental data of thermal conductivity. The study of Lu *et al*. (2007) aimed to develop a simple model to describe thermal conductivity in relation to the entire water content range. The predicted values presented a good agreement with measured thermal conductivity data from a wide range of soil textures from (Kersten 1949; Johansen 1977; Farouki 1981).

Several studies have outlined thermal conductivity is affected by numerous soils characteristics, such as: soils types, particle size distribution (PSD), mineral composition, bulk density, porosity, and degree of saturation/water content (Nakshabandi and Kohnke 1965; Usowicz *et al*. 2013; Zhang *et al*. 2015; Bertermann *et al*. 2018). Abu-Hamdeh (2003) evaluated thermal conductivity of sand, loam, sandy loam, and clayey loam with several densities and water contents. Results showed thermal conductivity increased with increased density and water content. Barry-Macaulay *et al*. (2013), utilized dry bulk density and degree of saturation as the main variables in different soil samples. They found that increases in density would cause increases in thermal conductivity as part of a linear relationship. Alrtimi *et al*. (2016) investigated the effect of degree of saturation on thermal conductivity of a sandy soil. Results indicated thermal conductivity increased with degree of saturation. Bruno and Alamoudi (2020) conducted model simulations on thermal conductivity results from tests performed by Alrtimi *et al*. (2016). Results showed increasing thermal conductivity with increasing dry bulk density and degree of saturation. Furthermore, Chen (2008) assessed thermal conductivity of four quartz sands and introduced a model of λ that accounts for the effect of saturation and porosity. Results revealed a linear regression between measured thermal conductivity and porosity, which was impacted by variation of degree of saturation of tested sands.

This paper presents the validity of five well-known thermal conductivity models by scrutinizing their applicability against diverse experimental datasets. This assessment utilizes published experimental measurements alongside measured values of λ from compacted samples of local soil under varying degrees of saturation. While previous studies have explored similar themes, this validation adds empirical rigor to existing studies by highlighting both contributions and limitations of these models. Furthermore, a particular emphasis is placed on the degree of saturation S_r and its influence on the thermal conductivity of soils.

2. Material and methods

2.1 Tested Material

The studied soil consisted of calcareous tufa, widely employed in Algeria for its favourable mechanical properties in pavements construction. Given its extensive abundance across different regions, calcareous tufa have great potentials as a building material for future applications. The soil was extracted from Beni-Saf (BS) region. According to USCS classification, the material was classified as a sandy-clay (SC) soil. Table 1 presents physical characteristics with maximum dry densities γ_{dmax} and optimal water contents w_{opt} of the Standard and Modified Proctor Optimum (SPO, MPO). Tested samples, labelled as (BS-01, BS-02), were produced based on these compaction parameters of (SPO, MPO) as follows: First, the pre-defined amount of w_{opt} was added gradually to a pre-oven dried mass. Next, the humid mix introduced in a double-piston cylindrical mould and statically compacted to the maximum dry bulk density γ_{dmax}. The objective of static compaction was to produce equivalent cylindrical samples of d=50 [mm], and h=100 [mm] with identical compaction parameters of Proctor tests (i.e., γ_{dmax}, w_{opt}). Next, samples subjected to various drying periods under laboratory conditions (Temperature close to 20 [°C] and HR =50-60 [%]) for 1, 7, 14, 21 and 28 days. To assess the degree of saturation of each set of samples after each drying period (Table 2), volumes and weights of samples was measured with a precision of 0.01 [mm] using a digital calliper and a balance.

Table 1 – Physical characteristics of tufa, in addition to compaction parameters of samples.

	Tufa		**Standards**
	Granulometry / [%]		
Gravel	12		XP-P94-041 NF P94-057
Sand	48		
Silt	21		
Clay	19		
	Plasticity / [%]		
Plasticity index (PI)	17		NF P94-051
Liquid limit (LL)	37		
	Proctor tests		
	Standard proctor (SPO)	Modified proctor (MPO)	
Code	BS-01	BS-02	NF P94-093
Maximum dry bulk density γ_{dmax} / [g. cm^{-3}]	1.79	1.98	
Optimal water content w_{opt} / [%]	15	12.7	
Degree of saturation S_r / [%]	78	90	

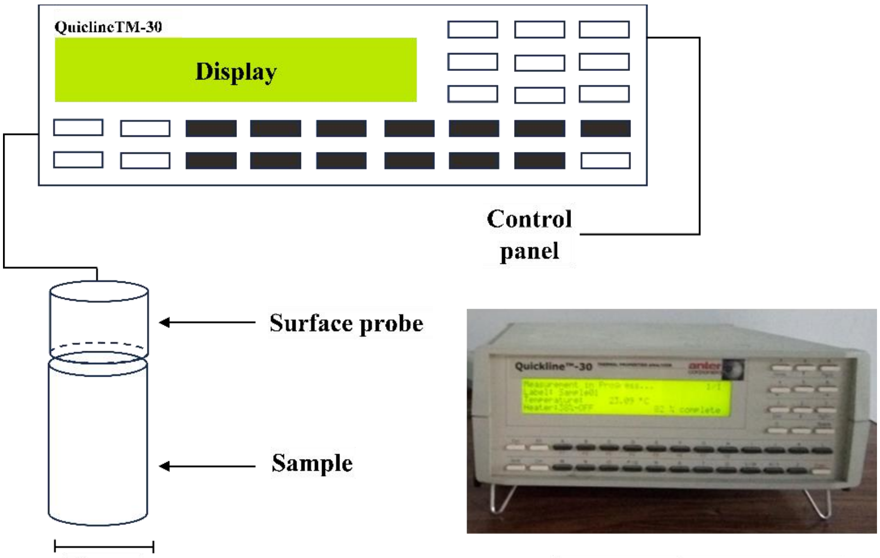
Table 2 – Bulk density and degree of saturation of compacted samples.

Code	**Drying time / [Day]**	**Dry bulk density / [g. cm^{-3}]**	**Degree of saturation / [%]**
BS-01	Immediately	1.797	78
	1	1.813	28
	7	1.800	10
	14	1.800	8
	28	1.815	6
BS-02	Immediately	1.930	90
	1	1.934	40
	7	1.949	9
	14	1.943	9
	28	1.944	8

2.2. Thermal Conductivity Testing Method

In general, thermal conductivity is measured through: steady-state methods (SSM) or transient methods (TM) (Yüksel 2016). The applicability and choice of measuring method depends on several parameters (Smith *et al*. 2013). Using (SSM) measurements, heat flow remains constant over time. These techniques are suitable only when the material under analysis is in complete equilibrium. Therefore, careful attention to sample preparation, methodology and required time to achieve necessary equilibrium is very important. These considerations play a key role in ensuring accuracy and reliability of (SSM) results (Jannot and Degiovanni 2018). Conversely, transient methods are time-dependent, i.e., heat flow may vary over time. The (TM) are particularly efficient in assessing thermal conductivity of moistened heterogeneous materials, as demonstrated by (Khushefati *et al*. 2022).

In this study, thermal conductivity λ was measured on samples after a drying period of 1, 7, 14, 21 and 28 days using the transient method via Quickline-30 device (Figure 1). Several studies employed and indicated the advantages of this equipment, such as straightforward operation, minimizing samples disturbances, and rapid measurements (Shin and Kodide 2012; Palumbo *et al*. 2016; Cheboub *et al*. 2020; Belaribi *et al*. 2024). This device was equipped with two types of probes: needle probe and surface probe. In this investigation, the surface probe was employed as it gives more accurate results comparing to the needle probe as reported by (Park and Vo 2015). Furthermore, the surface probe perfectly matched the in-contact surface of cylinder samples with diameter of 50 [mm].



(b) Photo of control panel

(a) Schematic diagram

Figure 1 – Experimental equipment used for thermal conductivity measurement.

The governing principle of thermal conductivity measurements, is based on the analysis of thermal response of samples to heat flow impulses. This later are induced by an electrical heating source (i.e., surface probe) in direct contact with samples' surface. Hence, thermal conductivity can be calculated according to Equation 1:

$$\lambda = \frac{Q}{4\pi(T_2 - T_1)} \ln\left(\frac{t_2}{t_1}\right) = \frac{2.3Q}{4\pi(T_2 - T_1)} \log\left(\frac{t_2}{t_1}\right) \quad (1)$$

where λ is thermal conductivity, [W.m^{-1}.K^{-1}], Q is heater power, [W], t_1 and t_2 represent initial and final measuring times, [s], T_1 and T_2 are initial and final measured temperatures, [K].

2.3. Experimental Data of λ from Literature

In addition to this study experimental results, thermal conductivity measurements of several soils from the study of Tarnawski *et al*. (2015) were examined. The database comprises 38 soils encompassing a variety of soil types predominantly classified as silt loam, sandy loam, loamy sand, and various clay types. These samples represent a broad spectrum of particle size distributions, with significant variability in bulk density (ranging from 0.98 to 1.71 [g/cm^3]) and porosity (ranging from 0.38 to 0.63). Notably, thermal conductivity measurements of tested soils reflect diverse degrees of saturation (S_r = 0, 0.1, 0.25, 0.5, 0.7, 1). In this context, the models under evaluation were assessed in terms of their ability to predict thermal conductivity across these varied soil types and conditions. Thereby providing further evaluation of their strengths and limitations in practical applications. Table 3 presents detailed physical properties of soils types, particle size distribution (PSD), bulk density, and porosity of selected soils from the work of Tarnawski *et al*. (2015).

Table 3 – Physical properties of documented soils (Tarnawski *et al*. 2015).

Soil No.	Code	Type	Particle size distribution / [%]			Bulk density / [g. cm^{-3}]	Porosity / [-]
			Clay	Silt	Sand		
1	NS-01	Silt loam	10.1	57.4	32.5	1.22	0.55
2	NS-02	Sandy loam	4.9	33.9	61.2	1.49	0.45
3	NS-03	Sandy loam	5.4	37.2	57.4	1.61	0.40
4	NS-05	Loamy sand	2.7	12.6	84.7	1.6	0.40
5	NS-06	Sandy loam	6.3	37.5	56.3	1.32	0.51
6	NS-07	Silt loam	11.8	66.5	21.7	1.2	0.57
7	PE-01	Loam	8	42.2	49.8	1.48	0.44
8	PE-02	Loam	9.1	39.5	51.4	1.54	0.42
9	PE-03	Loamy sand	2.7	13.9	83.5	1.57	0.41
10	NB-01	Silt loam	14.8	82.3	2.9	1.19	0.54
11	NB-02	Silt loam	16.6	83.4	0	1.12	0.56
12	NB-03	Silt loam	10.3	66.2	23.6	0.98	0.62
13	NB-05	Silty clay loam	32.8	67.2	0	1.25	0.54
14	QC-01	Sand	1.7	5.3	93	1.55	0.43
15	QC-02	Loamy sand	3.3	17.4	79.3	1.4	0.48
16	ON-01	Silt loam	7.6	55.9	36.5	1.54	0.43
17	ON-02	Silt loam	17.7	75.1	7.2	1.35	0.51
18	ON-03	Loamy sand	3.6	25.5	70.9	1.46	0.46
19	ON-04	Sand	1.1	9.6	89.3	1.68	0.39
20	ON-05	Sandy loam	7.4	36.9	55.7	1.71	0.38
21	ON-06	Loamy sand	2.2	14.1	83.7	1.53	0.44
22	ON-07	Silt loam	14.4	54.1	31.6	1.52	0.45
23	MN-01	Silt loam	13.7	69.1	17.3	1.21	0.55
24	MN-02	Silt loam	23.6	54.7	21.7	1.64	0.41
25	MN-03	Silt loam	21.3	75.8	2.9	1.01	0.63
26	MN-04	Loamy sand	3.3	15.4	81.3	1.43	0.47
27	SK-01	Silt loam	26.3	73.7	0	1.59	0.41
28	SK-02	Sandy loam	6.4	26.9	66.7	1.49	0.45
29	SK-03	Silt loam	14.9	83.3	1.8	1.27	0.53
30	SK-04	Loamy sand	3	14.2	82.8	1.56	0.42
31	SK-05	Sandy loam	4.8	27.6	67.6	1.47	0.45
32	AB-01	Silt loam	10	52	38	1.19	0.55
33	BC-01	Silty clay	41.8	58.2	0	1.34	0.51
34	BC-02	Silty clay	42	58	0	1.36	0.50
35	BC-03	Silty clay loam	29.6	70.4	0	1.33	0.51
36	BC-04	Silt clay	41.4	58.6	0	1.34	0.52
37	BC-05	Clay	33.2	66.8	0	1.3	0.53
38	BC-06	Silt loam	9.9	58.3	31.8	1.32	0.52

3. Theoretical and Empirical models

The studied models encompass: De Vries (1963), Johansen (1977), Farouki (1981), Hu *et al*. (2001), and Lu *et al*. (2007). The selection of these five models is based on their renowned simplicity and widespread acceptance within the research community.

3.1. De Vries Model 1963 (DVM)

De Vries model (1963) was based on Maxwell's equations for electrical conductivity of uniform spheres dispersed in a continuous fluid. In unsaturated soil, the soil solid particles and air voids assumed uniformly distributed in the continuous pore fluid (Haigh 2012). Consequently, thermal conductivity of soil was calculated as the weighted average thermal conductivities of soil components with consideration to its shape factors, Equation 2, (Dai *et al*. 2019):

$$\lambda = \frac{\sum_{i=0}^{n} R_i x_i \lambda_i}{\sum_{i=0}^{n} R_i x_i} \tag{2}$$

λ_i is thermal conductivity of each constituent: sand, silt, clay, and air which can be set, respectively as, 8.53, 2.93, 2.93, and 0.025 [W.m^{-1}.K^{-1}] (Tong *et al*. 2016). x_i is volume fraction of each constituent of soil. The ratio R_i describe the ratio of average thermal gradient of each constituent in relation to the continuous medium. R_i depend on several factors, such as particle size distribution, shape and relative position, and is given as follow (Equation 3):

$$R_i = \frac{1}{3}\left[\frac{2}{1+\left(\frac{\lambda_i}{\lambda_w}-1\right)f_i} + \frac{1}{1+\left(\frac{\lambda_i}{\lambda_w}-1\right)(1-2f_i)}\right], i{=}w,a \text{ (for air)}, s \text{ (for solid)}, \tag{3}$$

$\frac{\lambda_i}{\lambda_w}$ is the ratio of thermal conductivity of one constituent to that of the continuous medium (water). f_i denotes grain shape coefficients for the *i* component (Zhang and Wang 2017). f_i is taken as 1/3 if soil particles were assumed spherical. If soil particles were assumed ellipsoidal, it is calculated as follow (Equation 4):

$$\begin{aligned} &if\ i = s :\ f_i = 0.125\ ; \\ &if\ i = a :\ f_i = \begin{cases} 0.013 + 0.944\theta_s;\ where\ \theta_s S_r \leq 0.09 \\ 0.333 - (0.333 - 0.035)(1 - S_r);\ otherwise \end{cases} \end{aligned} \tag{4}$$

Where θ_s is soil's saturated water content, [cm^{-3}. cm^{-3}]

3.2. Johansen Model 1977 (JOM)

Johansen (1977) introduced the idea of normalized thermal conductivity using the concept of Kersten number K_e. In his work, thermal conductivity was established relying on thermal conductivities at both dry λ_{dry} and saturated conditions λ_{sat} according to Equation 5:

$$\lambda = \left(\lambda_{sat} - \lambda_{dry}\right) K_e + \lambda_{dry} \tag{5}$$

Johansen proposed the following function of K_e (Equation 6) (Wessolek *et al*. 2023):

$$K_e \simeq 0.7 \log S_r + 1.0 \quad (S_r > 0.05) \tag{6}$$

The thermal conductivity of dry soil (Equation 7) was expressed based on semi-empirical relationship as follows:

$$\lambda_{dry} = \frac{0{\cdot}137\rho_d + 64{\cdot}7}{\rho_s - 0{\cdot}947\rho_d} \tag{7}$$

ρ_d is dry bulk density of soil, [kg.m^{-3}], ρ_s is density of soil's solids, [kg.m^{-3}]. The thermal conductivity of the saturated soils λ_{sat} was established based on Equation 8:

$$\lambda_{sat} = \lambda_{solid}^{1-n}\lambda_{water}^{n} \tag{8}$$

n is soil porosity, λ_{water} represent the thermal conductivity of water, λ_{solid} represent the effective thermal conductivity of soil solids given by Equation 9:

$$\lambda_{solid} = \lambda_Q^{q}\lambda_o^{1-q} \tag{9}$$

λ_{solid} is calculated from quartz content q and its thermal conductivity λ_Q = 7.7 [W.m^{-1}.K^{-1}] as part of the total solids content, with: λ_o=2.0 [W.m^{-1}.K^{-1}] for $(q > 20\%)$; λ_o=3.0 [W.m^{-1}.K^{-1}] for $(q \leq 20\%)$

3.3 Farouki Model 1981 (FAM)

Farouki (1981) employed the equation initially suggested by Johansen (Equation 5). He deviated from using quartz content, and instead, he used all soil components to compute the thermal conductivity of soil's solids. Consequently, λ_{sat} was determined as follows (Equation 10):

$$\lambda_{sat} = \left(R_M\lambda_{M\_H} + R_{SORM}\lambda_{SORM_H} + R_G\lambda_G\right)^{1-w_s}\lambda_{water}^{w_s} \tag{10}$$

R_G, R_M, R_{SORM} represent volumetric fraction of gravel, mineral, and organic matter in soil, respectively. While, their thermal conductivities under dry conditions calculated as follows (Equation 11-12):

$$\lambda_{M\_H} = \frac{8.80(\%sand) + 2.92(\%clay)}{(\%sand + \%clay)} \tag{11}$$

(%sand) and (%clay) are gravimetric fractions of sand and clay. λ_{SORM_H} is set at 0.05. While λ_G as follows:

$$\lambda_G = 0.039\theta_s^{-2.2} \tag{12}$$

In addition, Kersten number K_e was simplified and calculated using Equation 13:

$$K_e = S_r + 1 \tag{13}$$

3.4 Hu et al. Model 2001 (HUM)

The model of Hu *et al*. (2001) was also based on Johansen's work with modified Kersten number K_e, expressed according to Equation 14:

$$K_e = 0.9878 + 0.1811\ln(S_r) \tag{14}$$

In addition, different thermal conductivities λ_{solid} and λ_{water} , respectively of, 3.35, 0.6 [W.m^{-1}.K^{-1}], were suggested to evaluate λ_{sat} and λ_{dry}.

3.5 Lu et al. Model 2007 (LUM)

Lu *et al*. validated the Côté and Konrad model, based on Johansen (Equation 5), and suggested a new function for K_e as follows (Equation 15):

$$K_e = exp\left\{\alpha\left[1 - S_r^{(\alpha-1.33)}\right]\right\} \quad (15)$$

Moreover, a linear relationship to estimate λ_{dry} was formulated (Equation 16):

$$\lambda_{dry} = -an + b \quad (16)$$

where α, a, and b are empirical parameters that could be determined through fitting measurements of soil porosity. Thus, following the formulations of Johansen, the thermal conductivity can be expressed as follows (Equation 17):

$$\lambda = [\lambda_{water}^{n}\lambda_{solid}^{1-n} - (b - an)]\exp[\alpha(1 - S_r^{\alpha-1.33})] + (b - an) \quad (17)$$

3.6 Model Evaluation Criteria

The accuracy of models in predicting thermal conductivity was assessed through comprehensive statistical analysis using the following metrics (Equation 18-22):

Relative error:

$$RE = \left|\frac{\lambda_{Exp,i} - \lambda_{Pre,i}}{\lambda_{Exp,i}}\right| \times 100 \quad (18)$$

Mean absolute error:

$$MAE = \frac{1}{N}\sum_{i=1}^{N} \left|\lambda_{Exp,i} - \lambda_{Pre,i}\right| \quad (19)$$

Root mean squared error:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \left(\lambda_{Exp,i} - \lambda_{Pre,i}\right)^2} \quad (20)$$

Index of agreement (Willmott's index):

$$IA = 1 - \frac{\sum_{i=1}^{N}\left(\left|\lambda_{Exp,i} - \lambda_{Pre,i}\right|\right)^2}{\sum_{i=1}^{N}\left(\left|\lambda_{Exp,i} - \bar{\lambda}_{Exp}\right| + \left|\lambda_{Pre,i} - \bar{\lambda}_{Exp}\right|\right)^2} \quad (21)$$

Coefficient of determination:

$$R^2 = \frac{\left(\sum_{i=1}^{N}(\lambda_{\mathrm{Pre},i} - \bar{\lambda}_{\mathrm{Pre}})\cdot(\lambda_{\mathrm{Exp},i} - \bar{\lambda}_{\mathrm{Exp}})\right)^2}{\sum_{i=1}^{N}(\lambda_{\mathrm{Pre},i} - \bar{\lambda}_{\mathrm{Pre}})^2\cdot\sum_{i=1}^{N}(\lambda_{\mathrm{Exp},i} - \bar{\lambda}_{\mathrm{Exp}})^2} \quad (22)$$

$\lambda_{\mathrm{Exp},i}$ is measured value of i^{th} data point, $\lambda_{\mathrm{Pre},i}$ is predicted value of i^{th} data point, $\bar{\lambda}_{\mathrm{Exp}}$ is the mean of measured values, $\bar{\lambda}_{\mathrm{Pre}}$: is the mean of predicted values. Each metric serves a specific purpose in evaluating the performance of models and contributes unique insights into their applicability and limitations. Relative Error (RE) measures accuracy of predictions in relation to experimental values, highlighting how close predictions are to actual measurements. This metric is essential for identifying models that may show low absolute errors but demonstrate inconsistent performance across different value scales. Mean Absolute Error (MAE) provides a straightforward assessment of average error in absolute terms, making it easy to interpret the overall model performance. However, MAE does not penalize larger errors, which can conceal significant issues in certain scenarios. In contrast, Root Mean Squared Error (RMSE) builds on MAE by assigning greater weight to larger errors, making it particularly useful. RMSE is expressed in the same units as data, offering a clear indication of typical magnitude of errors. Index of Agreement (IA), or Willmott's Index, assesses both bias and variability in predictions relative to experimental data. Thus, providing a comprehensive understanding of model performance across different data ranges. A higher value of IA suggests stronger agreement between measured and predicted values. Meanwhile, Coefficient of Determination R^2 quantifies the proportion of variance in experimental data explained by model's predictions, with a higher R^2 indicating better fit.

Utilizing all these metrics is essential, as each contributes complementary insights, ensuring balanced evaluation of accuracy and reliability across various soil types and conditions. While RE, MAE, and RMSE focus on error magnitude and prediction accuracy, IA offers a broader view of agreement between predicted and experimental values, and R^2 quantifies the goodness of fit and assesses model's ability to explain data variability.

4.Results and Discussion

4.1. Measured and Predicted Thermal Conductivity

Figure 2. Presents experimental values of λ in comparison to model predictions for samples BS-01 and BS-02 at initial state (immediately after compaction). Focusing on these values was essential for several reasons. Firstly, by starting at the initial state, we aimed to establish a clear understanding of how different models perform under standard conditions before exploring variations with changing degrees of saturation. Secondly, the initial state often serves as a baseline and an important input parameter in models' evaluation process. However, additional data points based on variations in degrees of saturation were included in subsequent sections to validate the applicability of models across multiple saturation levels. The observed patterns reveal models of Johansen (JOM), Lu *et al*. (LUM) and Hu *et al*. (HUM) consistently provided closer predictions to experimental values (denoted: EXP). Statistical analysis of (JOM), (LUM), and (HUM) revealed superior accuracy for samples BS-01 by indicating the lowest RE of 3.8, 4.5, 2.9 [%], respectively. Similarly, these three models indicated the lowest RE of 9.5, 9.3, 9.7 [%], respectively, for samples BS-02. In contrast, Farouki (FAM) and De Vries (DVM) models repeatedly presented lower accuracy for both samples by indicating higher relative errors, respectively, of 38, 106 [%] for BS-01 and 56, 89 [%] for BS-02. The findings of Figure 2 highlight that models of Johansen, Lu *et al*. and Hu *et al*. yielded, on average, to more accurate predictions of initial thermal conductivity λ in comparison to Farouki and De Vries models.

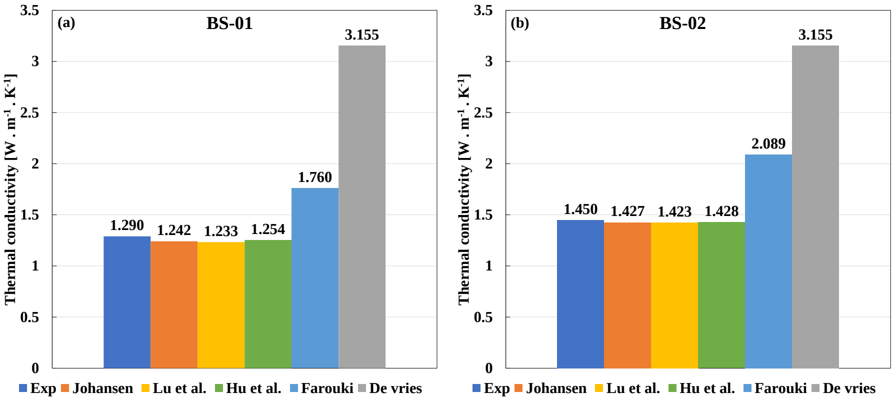


Figure 2 – Experimental and predicted thermal conductivity of samples BS-01, BS-02 (immediately after compaction).

Figure 3 compares the experimental values of thermal conductivity documented in Tarnawski *et al*. (2015), referencing for soils in table 3, with the corresponding predicted values from different models. The goal of this comparison is to ascertain the relative performances of the most accurate predictions models identified in previous section (i.e., Johansen, Lu *et al*. and Hu *et al*.). In Figure 3, each point represents a specific experimental value paired with its corresponding prediction. Ideally, points should align along the diagonal line (1:1 line), indicating perfect agreement between experimental and predicted values. As observed, predicted values from of Johansen and Lu *et al*. demonstrate strong agreement with experimental values, closely following the diagonal line (Figure 3.a,b). These models capture the overall trend, with an attributed attention to a few outliers and reasonably suggested to be the most performant models. Hu *et al*. present remarkable deviations from experimental data (Figure 3.c). A remarkable cluster of points is noted over the line (1:1) showing an overestimation of thermal conductivity. In contrast, of Farouki and De Vries (Figure 3.d,e) demonstrate significant discrepancies by indicating a major gap between experimental and predicted thermal conductivities. As observed, both models display a systematic overestimation of thermal conductivity across all ranges of soils.

Figure 3 also presents the different evaluation criteria for each model in predicting thermal conductivity. Regarding R^2, both Johansen and Lu *et al*. demonstrate higher values of 0.95, suggesting a strong ability to explain the variability in data. Followed by (HUM)with an R^2 of 0.83. Conversely, (FAM) and (DEV) present relatively lower R^2 values of 0.81 and 0.15, respectively. IA values measure the agreement between predicted and measured values, are generally high across the models of Johansen, Lu *et al*. and Hu *et al*. A strong agreement for (JOM) with IA of 0.972 and (LUM) with IA of 0.981. However, (LUM) stands out with the lowest MAE (0.106) and RMSE (0.151), thus, indicating superior accuracy and close agreement with measured values. (JOM) and (HUM)also presented good accuracy, situated in medium level, with relatively low MAE and RMSE. In contrast, Farouki and De Vries demonstrate deficient accuracy with higher errors, reflected in their MAE of 1.206 and 2.163, and RMSE of 1.243 and 2.363, respectively.

In conclusion, by examining data patterns of each model, the models of Johansen and Lu *et al*. can be highlighted. Conversely, moderate performance is observed for Hu *et al*. of similar predictions, yet with a marginally higher scatter. Regarding models of Farouki and De Vries, limitations in their predictions' accuracy can be remarked. These results can confirm the preceding findings in Figure 2.

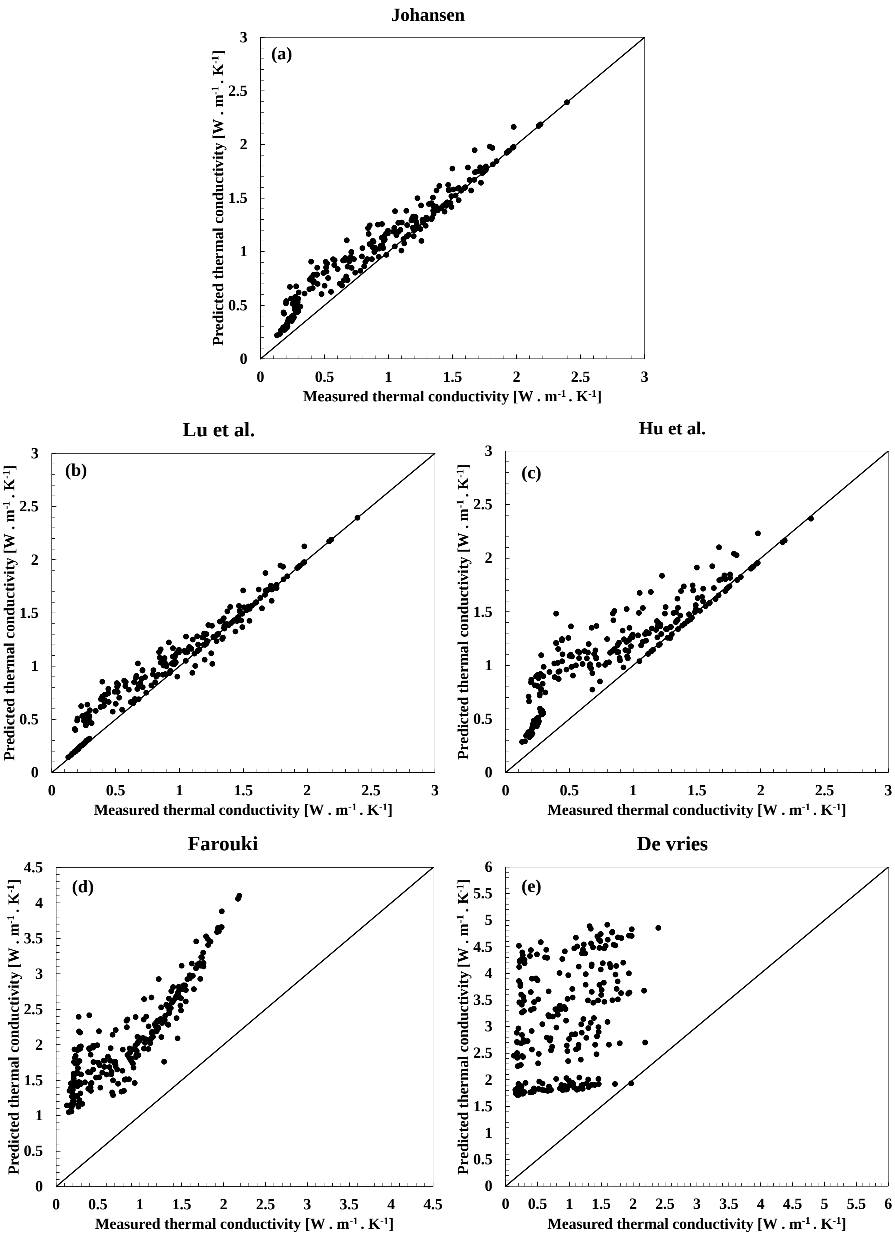


Figure 3 – Comparisons of predicted versus measured thermal conductivities for different soils. Diagonal line presents the 1:1 line.

4.2. Effect of Degree of Saturation

As mentioned earlier, the hydric state of soil represented by the degree of saturation, can be a significant factor influencing thermal conductivity. To delineate relationship between these complex variations of S_r and λ, the conceptual model proposed by Dong *et al*. (2015) can be suggested (Figure 4). In their analysis, it was stated that the full spectrum of thermal conductivity variations with saturation can be categorizes into several different regimes: pendular, funicular, and capillary. In pendular regime, delimited by ($0 \leq S_r \leq 0.2$), the thermal conductivity presented a slight increase. These minimal variations are explained by water infiltrations into dry soil, where thin films of water start enveloping soil particles. Hence, forming grain-water-grain connections through isolated inter-particle liquid bridges. As a result, this facilitates heat transfer through more conductive water phase, as thermal conductivity of water with a density close to 1 [g. cm^{-3}], is equal to 0.56 [W.m^{-1}.K^{-1}] (Bejan and Kraus 2003). Funicular regime ($0.2 \leq S_r \leq 0.8$), the thermal conductivity presented a considerable increase as more pore air is replaced by water until all the grain-water-grain connections are established. Where the thermal conductivity of air is approximately 20 times lower than water (Mňahončáková *et al*. 2006). Finally, in capillary regime ($S_r \geq 0.8$), additional water will lead to limited increases in thermal conductivity which almost remain constant as degree of saturation increases (Dong *et al*. 2015; Zhang *et al*. 2017).

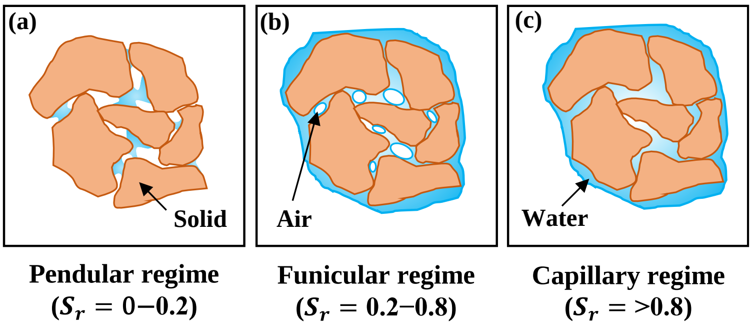


Figure 4 - Conceptual model of thermal conductivity variations with degree of saturation (pore-water distribution).

Figure 5 presents thermal conductivity λ variations with degree of saturation S_r of Beni-Saf samples. General decrease of experimental (Exp) values of λ with decreasing S_r was observed for both samples BS-01 and BS-02 from the beginning to end of drying periods. The highest values were 1.29 [W.m^{-1}.K^{-1}] for BS-01 at S_r = 0.77 [%] and 1.45 [W.m^{-1}.K^{-1}] for BS-02 at S_r = 0.9 [%]. The values of λ_{dry} of 0.67 [W.m^{-1}.K^{-1}] and 0.74 [W.m^{-1}.K^{-1}] for BS-01 and BS-02, respectively, were estimated by considering a linear variation trend of experimental data. Figure 5 also present the predicted values of λ for samples BS-01 and BS-02 resulting from the most accurate models identified in section (4.1): Johansen, Lu *et al*., and Hu *et al*. Results suggest that models of Johansen and Lu *et al*. have more reasonable trends and relatively closer accurate predictions. However, Hu *et al*.'s model remarkably overestimated thermal conductivity values.

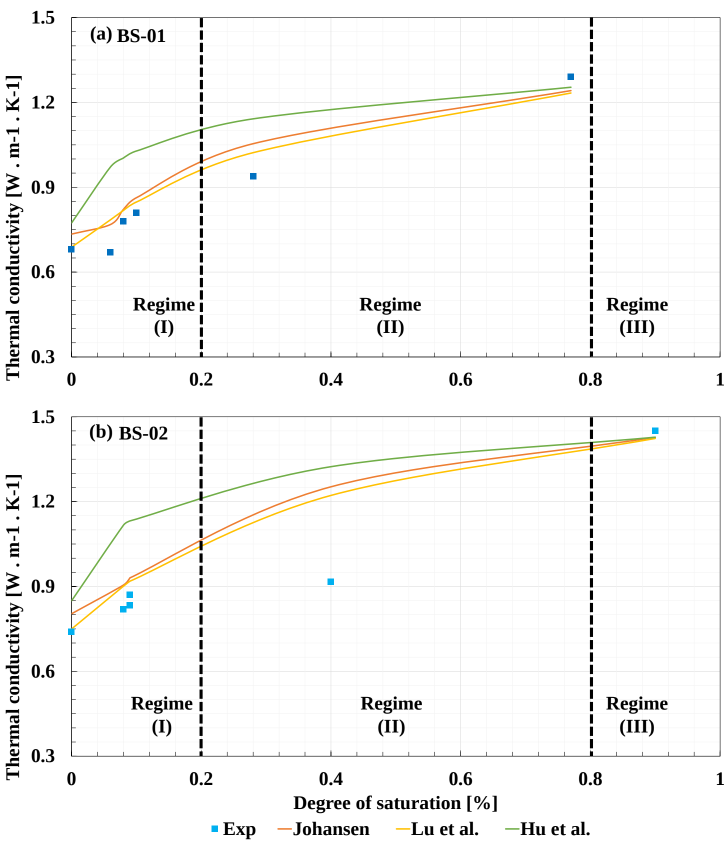


Figure 5 – Comparison of measured and predicted thermal conductivity against degree of saturation of Beni-Saf tufa: (a) BS-01, (b) BS-02.

It is worth highlighting that the above-mentioned regimes can be observed in figures 5. The pendular regime with lower degrees of saturation. The funicular regime presented by an important increase in λ values with increasing S_r. Eventually, the capillary regime, with relative low augmentation in thermal conductivity.

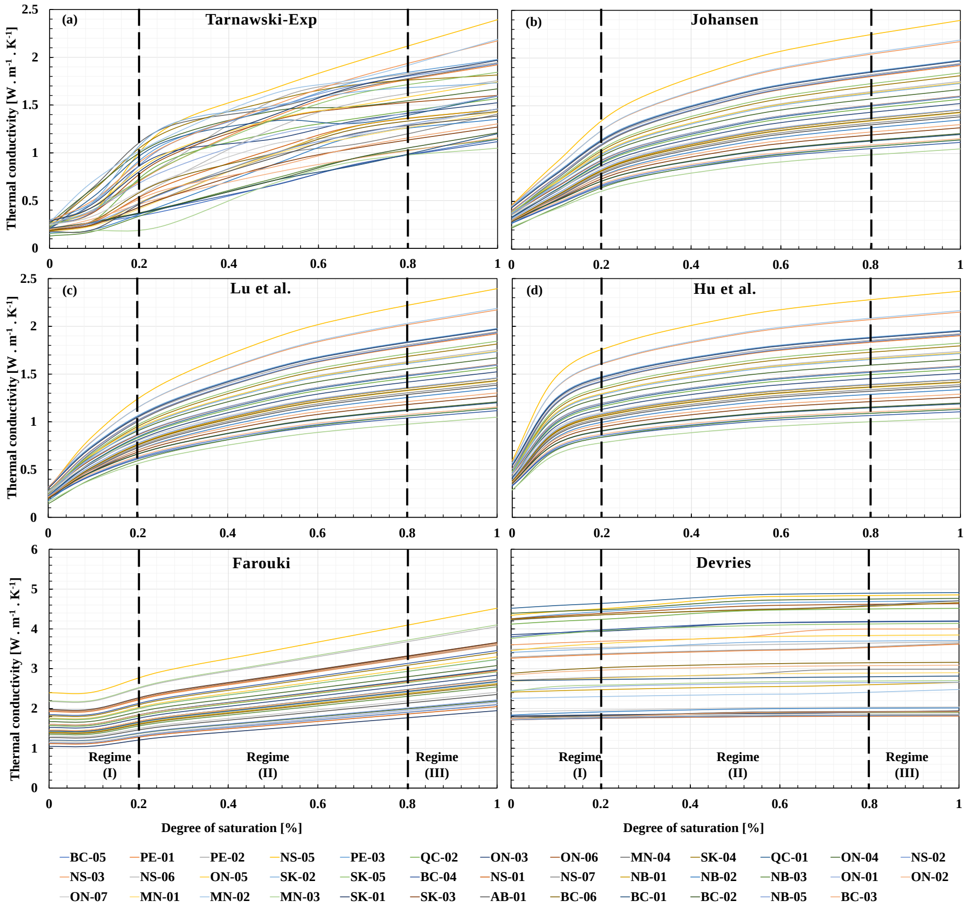


Figure 6 – Comparison of measured and predicted thermal conductivity against degree of saturation of soils by (Tarnawski *et al*. 2015).

Similarly, Figure 6 a presents comparison of measured and predicted thermal conductivity values against degree of saturation of different soils from Tarnawski *et al*. (2015). Figure 6.a presents experimental values of λ under varying. Across all types of soils, trends indicate increasing thermal conductivity with increasing degree of saturation. The lowest values were recorded as 0.13 [$W.m^{-1}.K^{-1}$] for NB-03 at S_r =0 [%] (dry state) and the highest values was 3.17 [$W.m^{-1}.K^{-1}$] for NS-04 at S_r = 100 [%] (saturation state). Figure 6.b, c, d, present predictions of λ as a function of S_r using models of Johansen, Lu *et al*. and Hu *et al*. Results clearly demonstrate increases in degree of saturation cause increases of thermal conductivity.

Table 4 presents the goodness of fit for different models at different degree of saturation through coefficients of determination R^2. At dry level (S_r=0), models of Johansen and Lu *et al*. demonstrate the highest values of R^2 of 0.93 and 0.99, respectively. At relatively dry state, (S_r =0.1), all models presented relatively low values of R^2. At the level (S_r =0.25), moderate values of R^2 were observed for models of Johansen, Hu *et al*., and Lu *et al*., while models of Farouki and De vries presented a noticeable improvement. At medium level (S_r =0.5), models of Johansen, Hu *et al*., and Lu *et al*. achieve a remarkable increase in coefficients of determination with (R^2 > 0.90). Additionally, Farouki model maintains reasonably high R^2 in contrast to De vries model. In subsequent levels of (S_r =0.7) and (S_r =1), higher values of R^2 were noticed for all models in the exception of De vries. In overall, Lu *et al*. model consistently demonstrates the highest R^2 values across different degree of saturation, followed by the models of Johansen and Hu *et al*.

Table 4 – R^2 values for predictions models at varying degree of saturation.

Model	R^2					
	S_r=0 (dry)	S_r=0.1	S_r=0.25	S_r=0.5	S_r=0.7	S_r=1 (sat)
Johansen	0.93	0.59	0.70	0.90	0.94	0.97
Hu *et al*.	0.81	0.42	0.70	0.91	0.94	0.94
Lu *et al*.	0.99	0.63	0.71	0.90	0.94	0.98
Farouki	0.02	0.30	0.66	0.89	0.95	0.98
De vries	0.05	0.34	0.68	0.56	0.44	0.42
	Global R^2		
	(I) - Pendular regime	**(II) - Funicular regime**	**(III) - Capillary regime**
Johansen	0.70	0.90	0.97
Hu *et al*.	0.66	0.80	0.94
Lu *et al*.	0.65	0.91	0.98
Farouki	0.28	0.87	0.98
De vries	0.20	0.39	0.42

Figure 7 illustrate the Global R^2 of predictions models across the three regimes: Pendular, Funicular, and Capillary. For pendular regime, all models demonstrate moderate and low performances with R^2 of 0.70, 0.65, and 0.66 for (JOM), (LUM) and (HUM), respectively. Moreover with R^2 of 0.28 and 0.20 for (FAM) and (DEV), respectively. Examining R^2 in funicular regime reveals interesting improvements of (JOM) and (LUM) with an R^2 of 0.90 and 0.91. However, relatively moderate performance observed for (LUM) and (FAM), with an R^2 of 0.80 and 0.87, respectively. The model of De vries consistently maintained the lowest R^2 of 0.39. Transitioning to capillary regime, Johansen and Lu *et al*. models continue to point out their robust predictive capability with an R^2 superior to 0.97. Interestingly, Hu *et al*. and Farouki models, despite their moderate performance in funicular regime, present R^2 values of 0.94 and 0.98 in this regime.

The findings suggest that models of Johansen and Lu *et al*. indicated good performs across all regimes. Farouki model has also its restrictions, regardless its relative simplicity. The models' accuracy may be influenced by variations in soil composition and structure. On the other hand, Limitations of De Vries model was observed across all regimes. This could be attributed to the assumption that solid particles and air voids uniformly distributed in continuous pore fluid (water), which might not be the case in real conditions of soils.

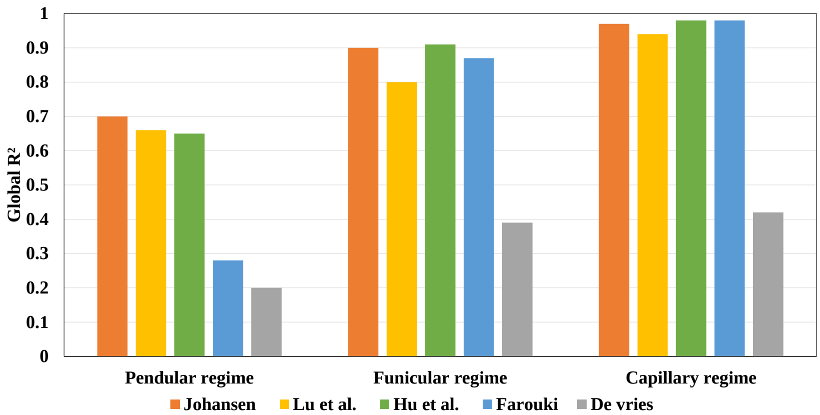


Figure 7 – Global coefficient of determination R^2 of predictions models across the three regimes.

An important point to note is that model of Hu *et al*. slightly overestimated thermal conductivity in compare to Johansen and Lu *et al*. models. These deviations can be explained by modifications made on Kersten number K_e. Consequently, different relation of (K_e – S_r) in each model can influence the relation (λ– S_r) in each soil. Figure 8 illustrated this trend by plotting the relation (K_e – S_r). As remarked, for the same degree of saturation, values of K_e in Hu *et al*. are higher than those from Johansen and Lu *et al* in pendular and funicular regimes ($0 \leq S_r \leq 0.8$). While the models of Johansen and Lu *et al* remain relatively close to each other. In capillary regime, all curves are closely juxtaposed. As results, the overestimation can be attributed to the proper selection of Kersten number, a factor significantly impacts the models' accuracy in prediction λ.

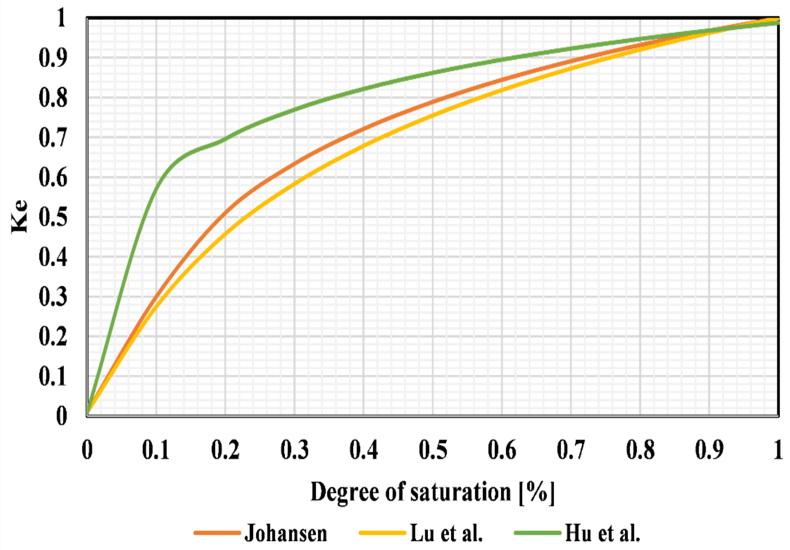


Figure 8 – Kersten number against degree of saturation for the models of Johansen, Lu *et al*., and Hu *et al*.

5. Conclusion

This paper presented a comprehensive study on thermal conductivity of soils by evaluating different results from models' prediction against experimental values. These experimental data were obtained from a local soil in Algeria, along to values from literature of 38 soils ranging from dry to saturated. Based on the reported findings, the main conclusions are:

- Transient method can offer an accurate method to measure thermal conductivity of the local soil (calcareous tufa) under study.
- Across all types of soils, results indicated increasing trends in thermal conductivity λ with increasing degree of saturation S_r.
- A total of five predictive models of thermal conductivity were examined: De Vries (1963), Johansen (1977), Farouki (1981), Hu *et al*. (2001), and Lu *et al*. (2007). A theoretical model and four empirical models based on Johanssen approach. This approach estimates thermal conductivity at any degree of saturation by interpolating between thermal conductivities of soils in both dry and saturated states, using Kersten number K_e.
- Notably, models of Johansen and Lu *et al*. demonstrated interesting accuracy in predicting λ across all degrees of saturation S_r. Hu *et al*. indicated relatively lower performance in compare with Johansen and Lu *et al* where it tended to overestimate λ. Model of Farouki started to provide accurate predictions as degrees of saturation approached higher values. Conversely, significant limitations in the predictive capability of De Vries model were observed.
- Analysis of Johansen, Lu *et al*., and Hu *et al*. models highlight considerable impact of the formulation and choice of Kersten number K_e on prediction accuracy of models.

In conclusion, this study underscores the importance of the thermal conductivity λ of soils across different degrees of saturation. Understanding the significance role of λ in numerous applications, enables us to consistently refine and validate predictive models, ensuring their reliability in real-world engineering scenarios. Moving forward, integrating these models with experimental data will further enhance their accuracy, paving the way for more efficient solutions.

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