



## **THECNICAL PAPER:**

### **SISFLOR: A COMPUTATIONAL SYSTEM TO DETERMINE THE OPTIMAL TREE BUCKING**

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dynamic programming

#### **ABSTRACT**

A continuously competitive forest market and tied to the demands for wood products promotes the study and development of applications that increase the revenue of the forest enterprises. At harvesting, the cutting pattern (forest assortment) in which the trees are traced is traditionally determined by the experience of the chainsaw operator without using any optimization technique, which may result in economic losses in relation to the commercialized products. In general, there are numerous distinct assortments that can be chosen and hardly processed by a brute-force algorithm. This is the forest assortment problem at the individual tree level with the objetice of maximizing the commercial values of the felled trees. stem-level bucking optimization problem. The aim is to maximize the sales value of harvested trees. Dynamic Programming (DP) is an efficient optimization technique to determine the optimum bucking tree as it significantly reduces the number of calculations to be made. Thus, the objective of this work was to develop a modern and intuitive computational system that is able to find the optimum tree stem bucking through DP to help companies over the bole tracing, therefore, characterizing itself as a tool that supports decision making. After the execution of the system, the optimum assortment is shown by sequentially detailing all products that should be removed from the analyzed bole as well as their respective volumes and revenue.

#### **Palavras-chave:**

colheita florestal  
padrão de corte  
otimização  
programação dinâmica

#### **SISFLOR: UM SISTEMA COMPUTACIONAL PARA DETERMINAR O SORTIMETO FLORESTAL ÓTIMO**

#### **RESUMO**

Um mercado florestal cada vez mais competitivo e atrelado às demandas por multiprodutos da madeira favorecem o estudo e o desenvolvimento de aplicações que busquem aumentar a receita dos empreendimentos florestais. Durante a colheita, o padrão de corte (sortimento) no qual as árvores são traçadas é determinado tradicionalmente pela experiência do motosserrista, sem o emprego de qualquer técnica de otimização, o que pode acarretar em perdas financeiras em relação aos produtos comercializados. Normalmente, existem inúmeros sortimentos distintos passíveis de escolha e dificilmente processados por um algoritmo de força bruta. Trata-se do problema do sortimento florestal em nível de árvore individual com o objetivo de maximizar o valor comercial das árvores abatidas. A Programação Dinâmica (DP) apresenta-se como uma técnica de otimização eficiente para determinar os sortimentos ótimos das árvores, pois permite reduzir significativamente o número de cálculos a serem feitos. Nesse sentido, o objetivo desse trabalho foi desenvolver um sistema computacional moderno e intuitivo capaz de encontrar os sortimentos ótimos via PD, a fim de auxiliar as empresas durante o traçamento dos fustes, caracterizando-se assim, como uma ferramenta de apoio à tomada de decisão. Após a execução do sistema, o sortimento ótimo é exibido detalhando-se sequencialmente todos os produtos que devem ser retirados do fuste analisado, assim como seus respectivos volumes e receita.

## INTRODUCTION

Investments in studies related to the cultivation, harvesting, transportation and commercialization of wood are essential for a company in the forestry sector to remain competitive and meet the demands of its customers (FOREST INDUSTRY ADVISORY COUNCIL, 2015). Therefore, a well elaborated planning of all stages of the forest production system is essential and must be done with care in order to ensure an efficient and sustainable production (GHOSH and SINHA, 2016).

The diversity of multiproduct produced by the forestry activity depends, among other factors, on the management plan to which the forest has been submitted. This plan is aligned with the needs of the consumer market, which, governed by the law of supply and demand, has great influence on which products will be produced, in addition to bringing dynamism to the enterprise (SILVA, 2018). As for forestry, it is necessary to have planned how the raw material will be mapped. The objective is to maximize financial yields by optimizing the use of available forest resources, considering production costs, market demands and marketing values for the multiproducts.

The cut pattern, or forest assortment, is the sequence of products that can be obtained from a given bole, which may be the same or different from each other (SILVA *et al.*, 2015a). Such products are generally logs of different lengths and diameters of the fine point (fpd). The cut pattern can vary, mainly depending on the products that will be removed and the dendrometric variables (diameters, height and volume) of each felled tree.

Determining the optimal assortment of trees is one of the main challenges faced by the managers, considering the individual characteristics of each bole. According to Arce *et al.* (2004), this task is performed almost exclusively by the chainsaw operator, based on his or her intuition, which can compromise the profitability of the forestry business.

This motivates researchers to develop analysis and simulation systems that help decision making, which is fundamental in determining the most profitable assortments. Studies in search of optimal

assortment solutions in the face of a combinatorial explosion with several alternatives are part of a specific category of Operational Research (OR) problems known as the Cutting and Packing Problem (PCE) (SILVA *et al.*, 2015 b). Such a problem still fits into a general class of problems known in the complexity of algorithms as NP-difficult (CORMEN *et al.*, 2012).

Considering that the current lack of modern information systems, compatible with the currently available technologies in the forestry area, aimed at solving the problem of forest assortment, the objective of this work was to implement a computational system for calculating the Forest assortment (SISFlor), capable of determining the optimum assortments of the boles in order to maximize the revenue according to the commercialization of harvested forest products.

The optimization method implemented in SISFlor was the Dynamic Programming (DP) (MEENAKSHI and RAWAT, 2017). The problem of forest assortment was modeled at the level of an individual tree in order to maximize the value of the stem (bucking-to-value) (LAROZE, 1999; NYBAKK *et al.*, 2008). The mathematical model used was based on Arce (2000). The built application also has a friendly and intuitive interface, in order to facilitate the configuration of the input data and the system parameterization.

## MATERIAL AND METHODS

The study of the forest assortment quantifies the multiproducts that can be obtained from a bole (KOHLENER, 2013). Thus, the trees were characterized by their Diameter at Breast Height (DBH), their height and a function that describes the diametric reduction from the base to the top (tapering function). The logs (marketable products), in turn, were defined by their length, fpd and sales value (R\$.m<sup>3</sup>). The height of the stump was placed as a configurable parameter in the system, although the value of 0.1 meters is commonly stipulated. SISFlor was implemented in the Java programming language (DEITEL, 2010) using the integrated environment of Netbeans development (DANTAS, 2011).

The total height of the trees, if it has not been

informed, is measured using the DBH and an adjusted hypsometric relationship and inserted in the system. The hypsometric relationship is a mathematical equation that enables to estimate the height of other trees using the information (DBH-height) collected from some trees (SHARMA and BREIDENBACH, 2015; ÇATAL and CARUS, 2018). The profile of the tree trunk was adjusted according to a tapering model (SILVA *et al.*, 2011), whose coefficients are informed by the user. The hypsometric relationship and the tapering model implemented are described below.

Hypsometric Relationship (PRODAN, 1965):

$$\widehat{Ht} = \frac{DAP^2}{(\beta_0 + \beta_1 * DAP + \beta_2 * DAP^2)} \quad (1)$$

Tapering Function (SCHOEPFER, 1966):

$$\widehat{a}_i = DBH * \left[ \beta_0 + \beta_1 * \left(\frac{h_i}{Ht}\right) + \beta_2 * \left(\frac{h_i}{Ht}\right)^2 + \beta_3 * \left(\frac{h_i}{Ht}\right)^3 + \beta_4 * \left(\frac{h_i}{Ht}\right)^4 + \beta_5 * \left(\frac{h_i}{Ht}\right)^5 \right] \quad (2)$$

Where,

$\widehat{Ht}$  = total estimated height (m).

DBH = diameter measured at 1.3 m from the soil (cm).

$\beta_i$  = coefficients of the models.

$\widehat{a}_i$  = diameter  $i$  estimated over the bole, and

$h_i$  = height  $i$  over the bole.

The total volume of the bole was calculated from the integral of the tapering function considering the height of the stump and the total height of the tree. In this sense, the system also allows to calculate the volume of any log from the initial and final heights of the bole informed by the user.

The information regarding the marketable products are the following: log length, minimum fpd, maximum fpd and value (in R\$.m<sup>-3</sup>). SISFlor allows the user to insert, remove and edit the multiproducts considered for cutting during harvest. The number of marketable products can be high, such as 23, 42 or even 106, as shown by the works of Santana (2013), Menon (2005) and Wang, Ledoux and Mcneel (2004), respectively. This highlights the importance of developing a

computational system capable of finding the optimal assortments, given the practical difficulties pointed out by chainsaw operators when determining such assortments through simple field experience with harvesting.

The optimization strategy adopted was the DP. It is an algorithmic technique that has been successfully applied in solving optimization problems originating from the most diverse areas, including PCE. This solution method consists of decomposing the original problem into a set of smaller and simpler problems to be solved, therefore, it is considered a technique to optimize multistage decision processes. The objective is to store the results of the subproblems already solved and, when they appear again, its results will be simply retrieved. Hence, this repetitive calculation is avoided, since the subproblems will be processed only once (PAPADIMITRIOU and STEIGLITZ, 1998; GOLDBARG and LUNA, 2005).

The implemented DP algorithm is based on the methodology of Arce (2000). From the base towards the tip of the bole, the algorithm continuously evaluates several stages, solving each one of them through recursive equations. Several cutting alternatives (states) in each of these stages are compared and only the most valuable is stored. By the principle of optimality, further decisions for the remaining stages will constitute an optimal policy, regardless of the policy adopted in the previous stages (TAHA, 2016).

The first step is to define all the cutoff points (useful numbers) in the bole from which the different assortment strategies will be evaluated. The pseudocode of the algorithm used to find the useful numbers is shown in Figure 1.

The recurrence equation used by Arce (2000) to maximize the gross revenue of the bole is given below. The order in which the different products are examined is established arbitrarily, without affecting the optimality of the result. In addition to the gross revenue accumulated in each of the useful numbers, other variables are also stored in order to control the remaining useful length, and the diameter of the bole corresponding to the height indicated by the state.

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1.  USEFUL NUMBERS ( $L; l[m]$ )
2.  START
3.       $\beta \leftarrow \min\{l[1]; l[2]; \dots; l[m]\};$ 
4.       $U[m+1] \leftarrow \{0\};$ 
5.       $F[m+1] \leftarrow \{0\};$ 
6.       $k \leftarrow m;$ 
7.      WHILE ( $k > 1$ ) DO IT
8.          FOR (each new log removed  $j \leftarrow 1, 2, \dots$  UP TO  $L - (u + j * l[k] \geq \beta)$ ) DO IT
9.               $F[k] \leftarrow \{u + j * l[k]\}$  where  $u \in U[k+1];$ 
10.         END-FOR
11.          $U[k] \leftarrow F[k] \cup U[k+1];$ 
12.         Remove  $U[k]$  repeated numbers;
13.          $k = k - 1;$ 
14.     END -WHILE
15.     PRINT  $U[1]$  with the list of the useful number;
16. FIM.

```

Source: CARNIERI *et al.* (1994) adapted by the author.

**Figure 1.** Algorithm pseudocode for the calculation of useful numbers

Where,

$L$  = length of the bole;

$m$  = quantity of the products;

$l_i$  = length of the product  $i$ ;

$\beta$  = the smallest length;

$U[k]$  = list of the all the useful numbers found in the study; and

$F[k]$  = useful numbers found from the  $k$  point.

$$\text{Max } F_k(x) = \max \{Pk + F_k(x - l_k), F_{k-1}(x)\}; \quad k = 2, 3, \dots, m \quad (3)$$

Where,

$x$  = cutting point on the bole.

$l_k$  = length of the  $k$  product.

$F_k(x)$  = accumulated gross revenue of the best product combination up to  $x$  length using the first  $k$  products.

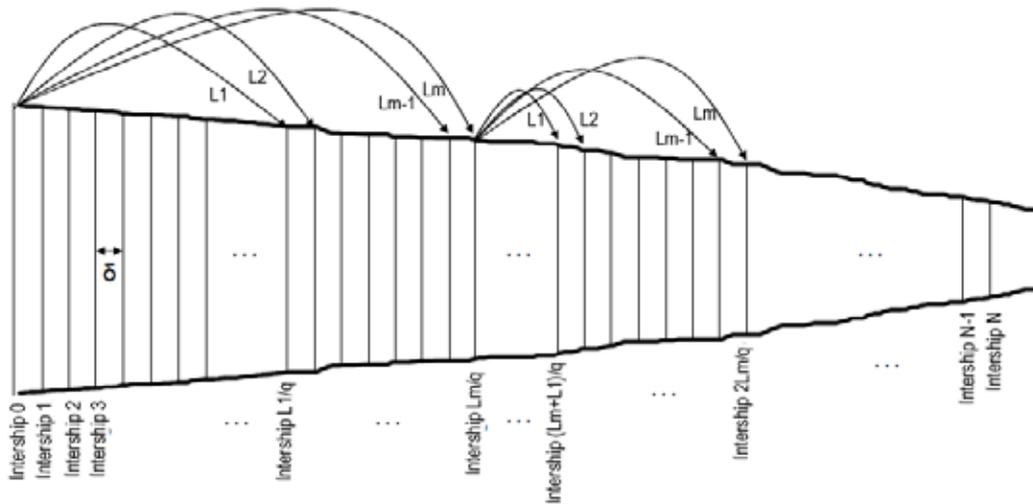
$P_k$  = gross revenue of the log of the evaluated  $k$  product, and

$F_k(x - l_k)$  = accumulated gross revenue of the best combination of the products obtained up to the length  $(x - l_k)$  using only the first  $k$  products.

Kivinen (2007) exemplifies (Figure 2) the formation of a network after the execution of the DP algorithm. In this case, the tree bole was divided into  $N$  segments, each with the length  $q$ . At each

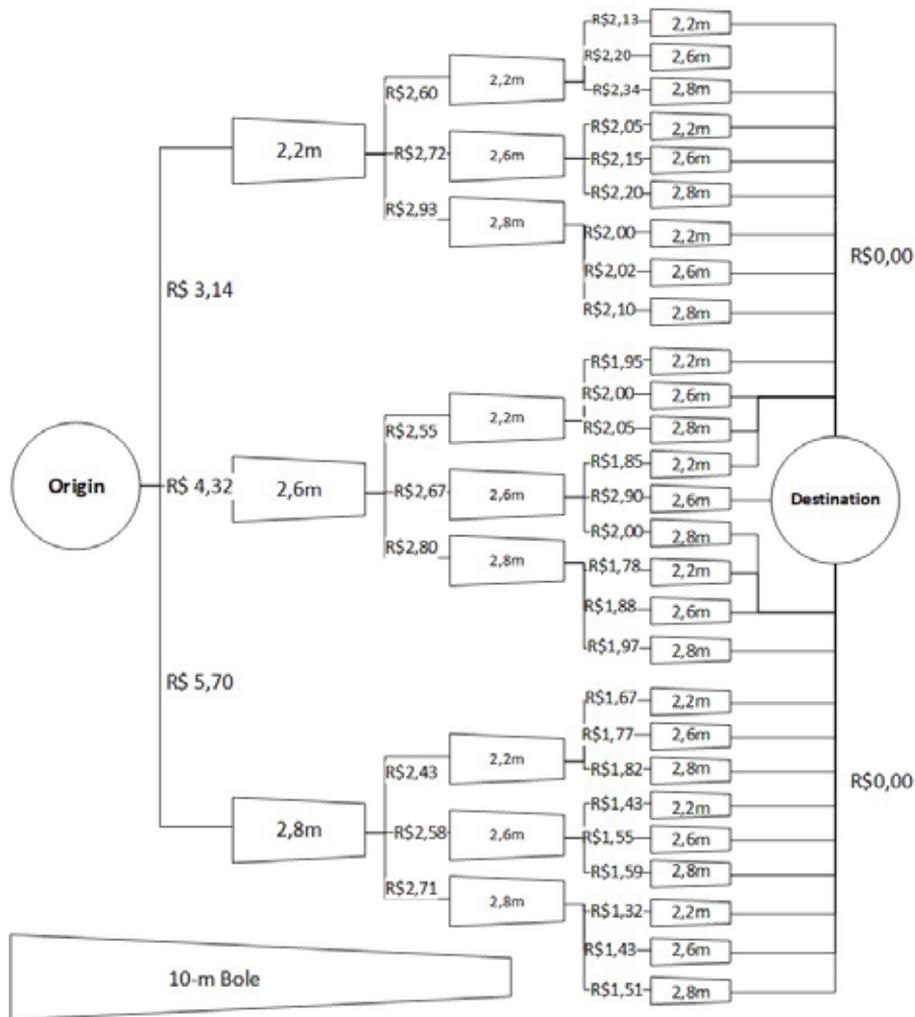
stage  $k$  ( $k = 0, 2, \dots, N$ ), all the  $m$  lengthable cutting products ( $L_1, L_2, \dots, L_m$ ) are tested and only the most valuable accumulated log combination is stored. In the end, the optimal assortment is the combination of products that have the highest value.

As an example, consider a fictitious 10-m bole to be drawn according to the sale of the following products (logs): 2.2 m for R\$ 58.00  $\text{m}^{-3}$ ; 2.6 m for R\$ 65.00  $\text{m}^{-3}$  and 2.8 m for R\$ 71.00  $\text{m}^{-3}$ . The network formed is illustrated in Figure 3. It is a targeted network with no cycles. The objective is to determine the optimal assortment among several alternatives, that is, the longest path between the origin (base of the bole) and the destination (tip of the bole). The last edges created with zero cost do not constitute a new log, but rather a necessary link to form the network and allow the processing of the solution method (SILVA, 2018).



Source: KIVINEN (2007) adapted by the author.

**Figure 2.** Representation of a network formed by the execution of the PD algorithm.



Source: SILVA (2018)

**Figure 3.** Example of a network formed by the problem of forest assortment in the bucking-to-value scenario.

**RESULTS AND DISCUSSION**

SISFlor, a computer system developed to optimize the forest assortment, presents a simple, intuitive and friendly graphic interface, which can also be used by users with little experience in forest harvesting. The input parameters are: DBH, height of the stump, height of the bole or the coefficients of the hypsometric relationship to calculate the height, the coefficients of the tapering function used and, finally, the characteristics of the products sold. The application screen can be seen using Figure 4.

The system calculates the optimal assortment per individual shaft. Thus, first, the user must inform the DBH and the height of the bole stump. If the height has been inventoried, it must also be informed. Otherwise, it must be calculated using the hypsometric relationship. By default, the Prodan (1965) model is suggested, in which only the values of the coefficients  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  are informed. However, the system also allows the choice of other models.

The total volume of the bole is calculated based on the DBH, total height and tapering function of the bole. It is worth noting that the useful volume of the bole is usually lower than its total volume since the commercialized products have specific

characteristics and may result in the failure of part of the bole stump. The tapering function configured as standard is the Schoepfer model (1966), therefore, it is necessary to inform the  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ ,  $\beta_4$  and  $\beta_5$  coefficients of the model. In addition, it is also possible to calculate the individual volume of the logs removed at any point of the bole desired by the system user. This functionality allows different cuts to be simulated, removing parts that have defects or anomalies such as knots, protuberances and resin bags from the bole. Figure 5 shows the parameterization of dendrometric variables of the SISFlor. The model coefficients were obtained from Menon (2005).

The products are inserted in the system one by one, informing their respective identifications (ID), length, minimum and maximum fpd and commercial value. If many products are commercialized, they can be loaded into the system from the information available in a text file. Figure 6 illustrates the registration of some products. All fields mentioned above are editable.

Finally, SISFlor informs the optimal assortment as a result from the available data and calculated through the DP. All products that must be removed from the bole are made available sequentially, from the base towards the top of the bole. In addition, the



Source: authors.

**Figure 4.** Screen of the developed system.

**Variáveis Dendrométricas**

DAP (Cm)

Altura (m)

Alt. do Toco (m)

Volume Total (m3)

**Relação Hipsométrica**

B0

B1

B2

**Calcular Volume da Tora**

Altura Inicial (m)

Altura Final (m)

Volume da Tora(m3)

**Função de Afilamento**

B0  B3

B1  B4

B2  B5

Source: authors.

**Figure 5.** System Parametrization.

**Inserir Multiprodutos**

ID	Comprimento	dpf Min	dpf Max	Valor (R\$/m3)
4.0	3.8	30.0	40.00	150.00
5.0	3.1	30.0	40.00	130.00
6.0	2.6	25.0	35.00	110.00
7.0	2.2	20.0	35.00	100.00
8.0	2	20.0	30.00	100.00
9.0	1.5	20.0	30.00	100.00

Source: authors.

**Figure 6.** Registration of the products.

amount collected from each log and its respective volume is reported, as well as the total amount collected from the bole and its useful length. The system internally uses 4 decimal places to calculate the value and volume of the logs supposedly produced, thus avoiding loss of precision in the rounding. The rounding strategy adopted was the

round-half-even (IBM KNOWLEDGE CENTER, 2014), often referred to in the literature as “banker rounding”. An example of an optimal assortment resulting from a bole with 30 meters in height and a 40-cm DAP is shown in Figure 7.

DP was proved to be an efficient and extremely fast method to find the optimal assortments, since

**Calcular Multiprodutos**

ID	Indice	Comprimento	Valor Tora
2	4.0	3.8	46.58
3	5.0	3.1	29.84
4	6.0	2.6	19.14
5	6.0	2.6	16.11
6	7.0	2.2	9.83
7	9.0	1.5	5.33

PD

Valor Total (R\$)

Comprimento Utilizado (m)

Source: authors.

**Figure 7.** Example of an optimal assortment calculated through DP.

all the processing was done in just one second. Meenakshi and Rawat (2017) state that, among the methodologies traditionally used in the literature to determine the optimal assortment of the stems, DP has been predominantly used. In general, DP is considered more efficient because it significantly reduces the number of calculations to be made.

## CONCLUSION

- A computational system denominated SISFlor was developed. This system is capable of quickly finding, through the DP, the optimal assortment of traced boles;
- DP proved to be an efficient method to solve the problem of forest assortment at the individual tree level as it obtained the optimal result in just one second in all tests performed.
- The system was considered by users as easy to be used, intuitive and with a modern graphic interface, which can be easily used to assist decision making during forest harvesting.

## REFERENCES

ARCE, J.E. **Um sistema de análise, simulação e otimização do sortimento florestal em função da demanda por multiprodutos e dos custos de transporte**. Curitiba, PR: UFPR, 2000. 136f. Tese (Doutorado em Ciências Florestais) – Universidade Federal do Paraná, Curitiba, 2000.

ARCE, J.E.; MACDONAGH, P.; FRIEDL, R.A. Geração de padrões ótimos de corte através de algoritmos de traçamento aplicados a fustes individuais. **Revista Árvore**, v.28, n.2, p.207-217, 2004.

CARNIERI C.; MENDOZA G.A.; GAVINHO L.G. Solution procedures for cutting lumber into furniture parts. **European Journal of Operational Research**, v.73, p.495-501, 1994.

ÇATAL, Y.; CARUS, S. A height-diameter model for brutian pine (*Pinus Brutia* Ten.) plantations in southwestern Turkey. **Applied Ecology and Environmental Research**, v.16, p.1445-1459, 2018.

CORMEN, T.H.; LEISERSON, C.R.; RIVEST, R.; STEIN, C. **Algoritmos: teoria e prática**. 3 ed. [s.l.]: Elsevier, 2012. 944p.

DANTAS, R. **NetBeans IDE 7 Cookbook**. 1 ed. [s.l.]: Packt Publishing, 2011.

DEITEL, H. **Java: como programar**. Prentice Hall Brasil, 2010.

FOREST INDUSTRY ADVISORY COUNCIL – DEPARTMENT OF AGRICULTURE. **Meeting future market demand Australia's forest products and forest industry**. Austrália, 2015. 45p.

- GHOSH, M.; SINHA, B. Impact of forest policies on timber production in India: a review. **Natural Resources Forum**, Nações Unidas, s.l., v.40, n.1-2, p.62-76, 2016.
- GOLDBARG, M.C.; LUNA, H.P.L. **Otimização combinatória e programação linear**: modelos e algoritmos. 2ed. Rio de Janeiro: Elsevier, 2005. 649p.
- IBM Knowledge Center – IBM. **Função round**. [s.l.]: IBM, 2014. Disponível em: <[https://www.ibm.com/support/knowledgecenter/pt-br/SSKM8N\\_8.0.0/com.ibm.etools.mft.doc/ak05380\\_.htm](https://www.ibm.com/support/knowledgecenter/pt-br/SSKM8N_8.0.0/com.ibm.etools.mft.doc/ak05380_.htm)>. Acesso em: 1 ago. 2018.
- KIVINEN, V.P. **Design and testing of stand-specific bucking instructions for use on modern cut-to-length harvesters**. Helsínquia, Finlândia: University of Kelsinki, 2007. 65f. Dissertação (Dissertação Acadêmica da Faculdade de Agricultura e Floresta) – University of Kelsinki, Helsínquia, 2007.
- KOHLER, S.V. **Evolução do afilamento do tronco e do sortimento em plantios de Pinus taeda nos estados do Paraná e Santa Catarina**. 2013. 88p. Dissertação (Mestrado em Engenharia Florestal) – Universidade Federal do Paraná, Curitiba, PR, 2013.
- LAROZE, A.J. A linear programming, tabu search method for solving forest-level bucking optimization problems. **Forest Science**, v.45, n.1, p.108-116, 1999.
- MEENAKSHI; RAWAT, K. **Dynamic Programming for Coding Interviews**: A Bottom-Up approach to problem solving. 1 ed. [s.l.]: Notion Press, 2017. 142p.
- MENON, M.U. **Meta-heurísticas na otimização do sortimento florestal**. Curitiba, PR: UFPR, 2005. 119f. Tese (Doutorado em Ciências Florestais) – Universidade Federal do Paraná, Curitiba, 2005.
- NYBAKK, E.; BIRKELAND, T.; FLÆTE, P.O.; FINSTAD, K. From a bucking-to-value to a bucking-to-demand system in Norway: A case study in forests with varying growth conditions. In: Proceedings of the 51st International Convention of Society of Wood Science and Technology, 2008. Concepción, Chile. **Anais...** Concepción: Chile, 2008.
- PAPADIMITRIOU, C.H.; STEIGLITZ, K. **Combinatorial Optimization**: Algorithms and Complexity. [s.l.] Dover Publications, 1998.
- PRODAN, M. **Holzmesslehre**. Frankfurt am Main: J. D. Sauerländer's Verlag. p.44. 1965.
- SANTANA, C.J.O. **Traçamento e alocação de toras de eucalipto por geração de colunas e programação dinâmica**. Curitiba, PR: UFPR, 2013. 96f. Tese (Doutorado em Ciências Florestais) – Universidade Federal do Paraná, Curitiba, 2013.
- SCHÖEPFER, W. **Automatisierung des massen, sorten and wertberechnung stenender waaldbestande schriftenreihe bad**. Berlin: Wurt-Forstl, 1966.
- SHARMA, R.P.; BREIDENBACH, J. Modeling height-diameter relationships for Norway spruce, Scots pine, and downy birch using Norwegian national forest inventory data. **Forest Science and Technology**, v.11, n.1, p.44-53, 2015.
- SILVA, F.da; CORTE, A.P.D.; SANQUETA, C.R. Equações de afilamento para descrever o volume total do fuste de Pinus caribaea var. hondurensis na região do Triângulo Mineiro. **Scientia Forestalis**, v.39, n.91, p.367-376, 2011.
- SILVA, R.F.; MONTES, D.P.; KAMPKE, E.H.; SILVA, G.F. Otimização do Sortimento de um Povoamento de *Eucalyptus Grandis* Através de um Algoritmo Guloso. In: III Congresso Brasileiro de Eucalipto, 2015, Vitória (ES). **Anais...** Vitória: III Congresso Brasileiro de Eucalipto, 2015 a.
- SILVA, R.F.; MONTES, D.P.; SILVA, G.F. Calibração de uma heurística de construção por partes para otimizar o sortimento florestal em plantios de *Pinus taeda L.* In: Simpósio brasileiro de pesquisa operacional, 47., 2015, Porto de

Galinhas. **Anais...** Porto de Galinhas: UFPE, 2015 b. p.592-602.

SILVA, R.F. **Modelagem e avaliação de diferentes métodos de otimização do sortimento florestal**. Jerônimo Monteiro, ES: UFES, 2018. 180f. Tese (Doutorado em Ciências Florestais) – Universidade Federal do Espírito Santo, Jerônimo Monteiro, 2018.

TAHA, H.A. **Operations Research: an introduction**. 10 ed. Estados Unidos: Pearson, 2016. 848 p.

WANG, J.; LEDOUX, C.B.; MCNEEL, J. Optimal tree-stem bucking of northeastern species of China. **Forest Products Journal**, Estados Unidos, v.54, n.2, p.45-52, 2004.