







NUMERICAL SIMULATION APPLIED TO MILK COOLING

Renan Pereira Rezende*¹ , Ednilton Tavares de Andrade¹ , Jefferson Luiz Gomes Correa²  & Ricardo Rodrigues Magalhães³ 

1 - Federal University of Lavras, Department of Agricultural Engineering, Lavras, Minas Gerais, Brazil

2 - Federal University of Lavras, Department of Food Science, Lavras, Minas Gerais, Brazil

3 - Federal University of Lavras, Department of Automatics, Lavras, Minas Gerais, Brazil

Keywords:

Mathematical modelling
Finite element method
Milk cooling tank

ABSTRACT

A model is a representation of a real system that can be analysed and yield predictions under different operating conditions. The aim of this study was to model a milk cooling tank that cools milk to 4 °C to preserve its quality after milking at the farm. The model was developed and simulated using the software Ansys for finite element analysis. The results from the simulations were compared to experimental data. The model simulated milk cooling in the tank with an error lower than 2%, which is considered acceptable for numerical simulations. In other words, the model satisfactorily represents the real system. Thus, alternatives can be directly tested in the computational model to improve and optimise the milk cooling process and to better use the system without actually implementing them in the real system.

Palavras Chave:

Modelagem matemática
Método dos elementos finitos
Tanque de resfriamento

SIMULAÇÃO NUMÉRICA APLICADA A REFRIGERAÇÃO DE LEITE

RESUMO

Um modelo é a representação de um sistema real que permite analisar e prever diversas condições de funcionamento. O objetivo deste trabalho visa a modelagem de um tanque de resfriamento de leite até 4 °C, de modo a manter a qualidade após a ordenha em uma propriedade rural. O modelo do sistema foi desenvolvido e simulado no software ANSYS via análise de elementos finitos. Os resultados das simulações foram comparados com dados experimentais. O modelo desenvolvido simulou o resfriamento do leite no tanque com um erro menor que 2%, o que pode ser considerado aceitável em se tratando de simulações numéricas. Isso significa que o modelo desenvolvido pode ser utilizado para representar satisfatoriamente o sistema real. Com isso, testes de alternativas para melhorar e otimizar o processo de resfriamento de leite, bem como uma melhor utilização do sistema, podem ser realizadas diretamente no modelo computacional, sem a necessidade de se implementar previamente no sistema real.

INTRODUCTION

Brazil produced 35.3 billion litres of milk in 2018, and its annual growth rate will be between 2.1 and 2.9% in the next 10 years (MINISTÉRIO DA AGRICULTURA, PECUÁRIA E ABASTECIMENTO, 2018). Efforts have been concentrated on improving the production process and on accelerating the incorporation of technologies, especially in medium and large dairy farms (EMBRAPA, 2011).

This sector has also tightened product quality requirements by adopting measures aimed at improving Brazilian milk standards. The search for quality seeks not only gains in the productivity and profitability of dairy farmers and the industry but also guaranteed food quality and safety and, mainly, consumer health (EMBRAPA, 2011). As the consumption of milk and dairy products is linked to principles of nutrition and health, ensuring the safety and quality of these products is essential for the industry.

Thus, the focus has been on dairy farmers because that is where the quality process starts. The cooling temperature of milk is directly related to the preservation of its quality. According to Normative Instruction No. 62 (2011), the cooling temperature should be 7 °C at most on the farm, and the milk should reach this temperature within 3 hours after milking. Even at this temperature, the bacterial load may still increase during storage. However, if the temperature remains lower than 4 °C, the bacterial count practically does not change in 48 hours (CLÍNICA DO LEITE, 2016).

RAGSDALE (2009) stated that a computational model is a representation of a real-world problem or system and that when using a model, decision alternatives can be analysed before a specific plan is chosen for implementation in the real system. PENG *et al.* (2017) mention that when using a model, improvements can be proposed and alternatives can be tested and optimised to predict the best approach without performing physical experiments. Computational models are used for heat transfer analysis in a wide range of applications,

as observed in Atangana (2016), Sheikholeslami and Ganji (2014), and Sheikholeslami *et al.* (2015).

With the significant increase in computational capacity, the finite element method (FEM) has been developed and widely used for its high reliability and accuracy of results (NIMDUM *et al.*, 2015). The FEM is widely used for the thermal analysis of systems (ZHAO, 2014).

This study was performed because numerical simulation has never been applied to processing in the dairy industry in similar research.

Thus, this study aims to model a milk cooling tank and to compare the simulation data to experimental results.

MATERIALS AND METHODS

At the farm where the study was conducted, in the municipality of Ingaí – Minas Gerais (MG), Brazil, all milk from both daily milkings is stored in a bulk tank until being hauled to the dairy industry. The bulk tank has a maximum capacity of 1650 L, with a height of 96 cm and a base diameter of 152 cm. The coating material is stainless steel, with inner and outer layers and with (2-cm-thick) expanded polystyrene between the two layers. The tank cools the milk through a heat exchange system, where chlorodifluoromethane gas (also known as R22) is circulated between the bottom of the tank (heat loss), and the outside (cooling), resuming the cycle. Temperature homogenisation is provided by a rectangular single-blade impeller (10 cm height and 60 cm length). Spinning at a constant speed of 2.07 m s⁻¹, the stainless-steel paddle stands 10 cm from the bottom of the tank supported by a bar vertically fixed to the lid of the tank. The vertical bar was disregarded in the simulations.

Figure 1 shows one of the geometries used to model milk in the bulk tank, with the milk volume and blade impeller.

The milk temperatures inside the tank were measured using the internal temperature monitoring system of the tank, consisting of a probe located 10 cm above the bottom of the tank on the sidewall.

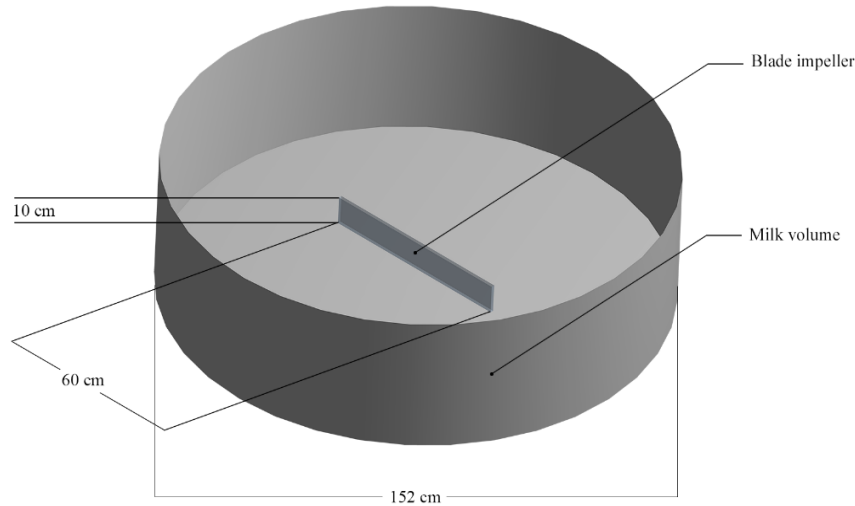


Figure 1. Geometry used to represent the milk in the tank and the impeller. The cylinder represents the milk volume, with a diameter of 152 cm and a height varying as a function of the milk volume: 16.8 cm for 305 L; 0.303 m for 550 L; and 0.427 m for 775 L

The heat fluxes assessed when cooling specific milk volumes on different days were calculated using Equation 1.

$$\varphi = \frac{Q}{s \cdot A} \quad (1)$$

where φ is the heat flux ($J \cdot s^{-1} \cdot m^{-2}$); Q is the amount of heat (J); s is the time (in seconds); and A is the area of contact between milk and the bottom of the tank (m^2).

The computational model developed and used to simulate the milk cooling process assumes the following working hypotheses: a transient state (as a dynamic system over time), a three-dimensional flow, and an isothermal system. As a boundary condition, a non-slip condition was considered for the boundaries with solid surfaces.

The momentum conservation equation used by the model is shown below (Equation 2):

$$\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{\tau}) + \rho \vec{g} \quad (2)$$

where ρ is the density; p is the static pressure; $\bar{\tau}$ is the stress tensor; and \vec{g} is the gravitational force.

The k- ϵ model was used to predict the turbulence inside the system during the simulation and is given by Equation 3:

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left[\frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + 2 \mu_t E_{ij} E_{ij} - \rho \epsilon \quad (3)$$

where ρ is the density; k is the turbulent kinetic energy; u_i is the component of the velocity in the corresponding direction; E_{ij} is the component of the deformation rate; and μ_t represents the turbulent viscosity, where $\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$;

The energy dissipation ϵ inside the cooling system is given by Equation 4:

$$\frac{\partial}{\partial t} (\rho \epsilon) + \frac{\partial}{\partial x_i} (\rho \epsilon u_i) = \frac{\partial}{\partial x_j} \left[\frac{\mu_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right] + C_{1\epsilon} \frac{\epsilon}{k} 2 \mu_t E_{ij} E_{ij} - C_{2\epsilon} \rho \frac{\epsilon^2}{k} \quad (4)$$

where u_i is the component of the velocity in the corresponding direction; E_{ij} is the component of the deformation rate; μ_t is the turbulent viscosity, and $\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$; and the following are constants: $C_\mu = 0.09$; $\sigma_k = 1$; $\sigma_\epsilon = 1.30$; $C_{1\epsilon} = 1.44$; and $C_{2\epsilon} = 1.92$.

The energy that governs the model is given by Equation 5:

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{v}(\rho E + p)) = \nabla \cdot (k_{\text{eff}} \nabla T) + S_h \quad (5)$$

where **E** is the energy; **T** is the temperature; **k_{eff}** is the effective thermal conductivity; and **S_h** is the heat source term.

Computer simulations were performed using the commercial software Ansys version 14.2. The tank was considered thermally insulated on the

side and bottom walls. To model the system, only the milk volume was considered in the simulations, that is, a cylindrical volume based on a diameter of 152 cm and a variable height, which was calculated based on the volume occupied in a given situation.

The meshes of the geometries used for each milk volume modelled in this study have tetrahedral elements and are shown below, in Figure 2.

Table 1 shows the numbers of nodes and elements for the meshes of the three milk volumes tested in this study.

The physical properties considered in modelling the system are outlined in Table 2.

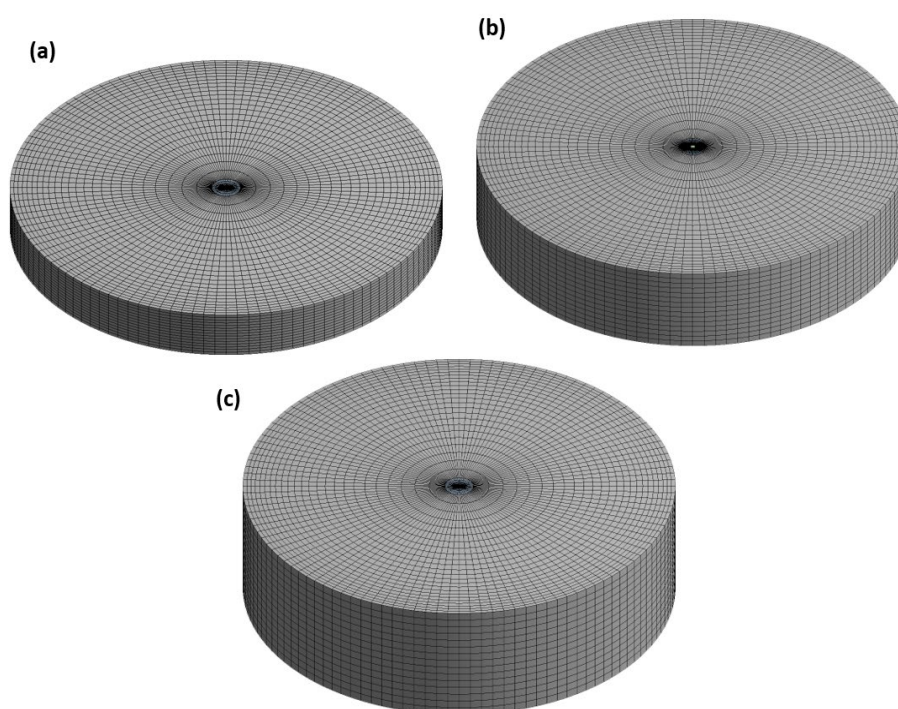


Figure 2. Meshes used: (a) 305 L; (b) 550 L; (c) 775 L

Table 1. Numbers of nodes and elements of the meshes for the three milk volumes tested in this study (305, 550, and 775 litres)

Volume (litres)	Nodes	Elements
305	54976	48555
550	57136	50175
775	58816	51435

Table 2. Physical properties of the milk and stainless steel considered in the modelling

Material	Density (kg.m ⁻³)	Specific Heat (J.kg ⁻¹ .°C ⁻¹)	Thermal Cond. (W.m ⁻¹ .°C ⁻¹)	Viscosity (kg.m ⁻¹ .s ⁻¹)
Milk ¹	1032	3890	0.53	0.002
Stainless steel ²	7955	510	15.5	-

Fonte: 1 - CHANDAN, 1997; 2 - AZO MATERIALS, 2001

RESULTS AND DISCUSSION

The results from the observations of the milk volume in the tank, initial and final temperatures, total time, and heat flux at the bottom of the tank

during cooling are outlined in Table 3.

Figures 3, 4, and 5 show the temperature distribution in the tank at 1800 seconds (30 minutes) after starting the simulation, for the volumes of 305 L, 550 L, and 775 L, respectively.

Table 3. Observations on different days with different milk volumes of in the tank

Observation	Volume (m ³)	Temperature (°C)		Time (min)	Heat flux ϕ (W m ⁻²)
		Initial	Final		
1	0.305	22.1	4.0	66.29	-3095.28
2	0.550	23.2	4.0	92.35	-4250.60
3	0.775	23.6	4.0	169.30	-3335.06

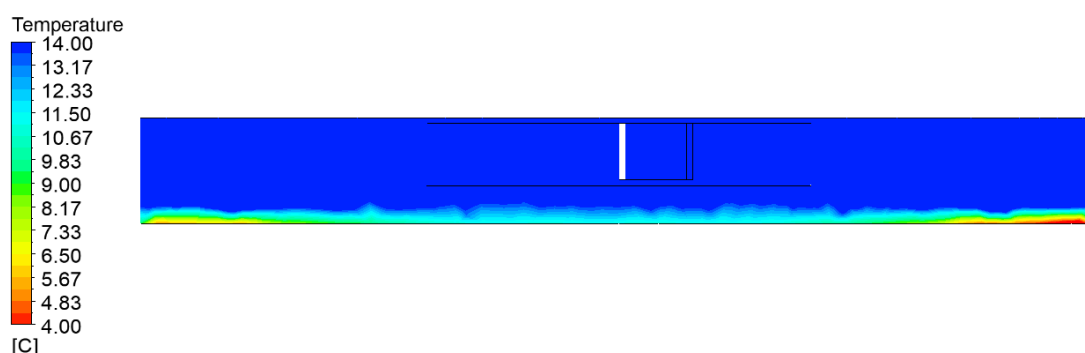


Figure 3. Cross-section of the temperature distribution inside the tank for the volume of 305 L after 30 minutes of simulation

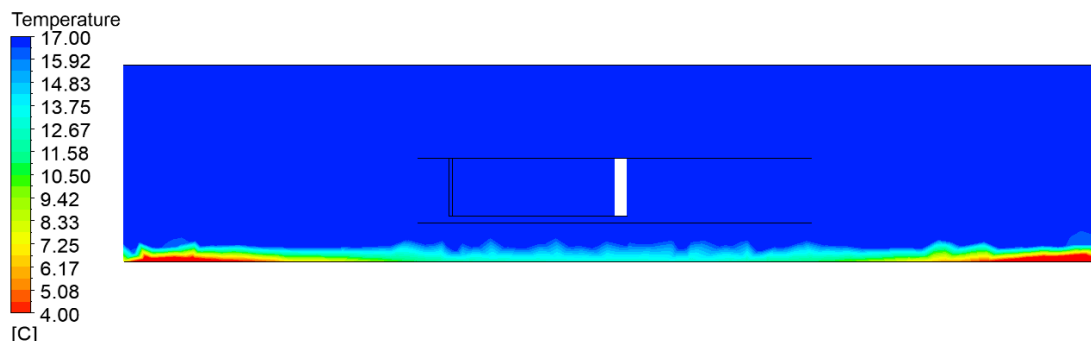


Figure 4. Cross-section of the temperature distribution inside the tank for the volume of 550 L after 30 minutes of simulation

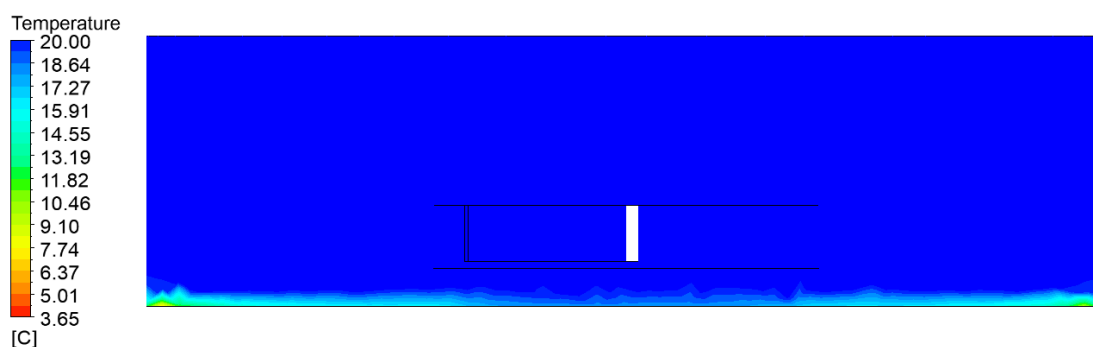


Figure 5. Cross-section of the temperature distribution inside the tank for the volume of 775 L after 30 minutes of simulation

The variation in the mean temperature as a function of time was also plotted (Figure 6) using the values observed in each volume (0.305 m³, 0.550 m³, and 0.775 m³).

Table 4 outlines the time required to cool the milk to 4 °C in the real system and the time required to cool the milk to 4 °C through simulation. The absolute percentage error was calculated for each observation of each simulated milk volume using Equation 7.

$$\text{Erro} = \left| \frac{\text{simulation time} - \text{real time}}{\text{real time}} \right| \quad (7)$$

The overall mean absolute error calculated for the three milk volumes simulated was 1.20%. According to MOHAPATRA and RAO (2005), deviations of up to 10% between the real values

and values determined by a computational model simulation are considered satisfactory and indicate that the computational model can be used to represent the real situation.

Thus, the model developed in this study performs satisfactorily and can be used to represent the real milk cooling tank under specific conditions.

CONCLUSION

- A computational model was built and simulated to represent a milk cooling tank used to cool milk to 4 °C within 3 hours. The model was tested for three volumes in the tank, with different temperatures at the beginning of the process. The mean absolute percentage error between the time elapsed in the real situation and the equivalent time in the simulation for the simulated milk volumes was 1.20%.

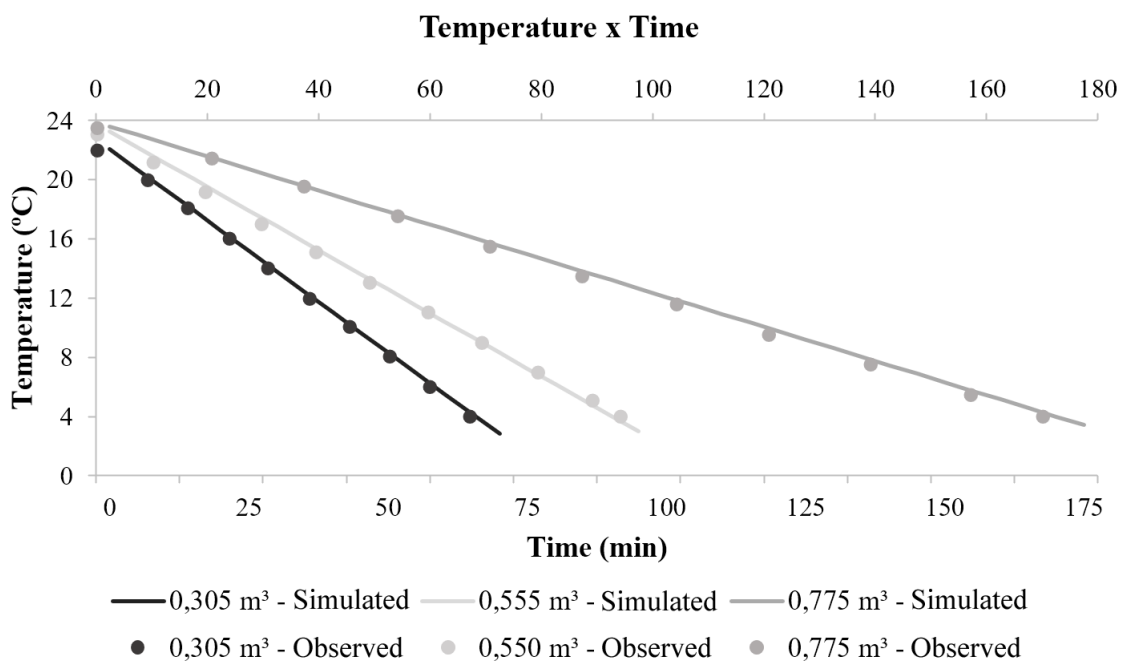


Figure 6. Mean temperature (°C) as a function of time for different mill volumes

Table 4. Real time, simulation time, and mean absolute percentage error for each simulated volume

Simulation	Volume (m ³)	Time (min)		Mean absolute error (%)
		Real	Simulated	
1	0.305	66.29	65.71	0.87
2	0.550	92.35	90.22	2.31
3	0.775	169.30	170.03	0.43
Mean				1.20

- The findings show that the model can be used to represent the real milk cooling tank within an acceptable error without having to directly test alternatives in the real system. Any changes introduced to optimise the process, whether in the efficiency of the cooling system or in the energy efficiency of the equipment, among others, can be initially tested in the computational model, which provides a good estimate of how the real system would behave under such changes, thereby increasing the efficiency whilst reducing costs and streamlining the experimentation process.

AUTHORSHIP CONTRIBUTION STATEMENT

RESENDE, R.P.: Conceptualization, Data curation, Investigation, Resources, Writing – original draft; ANDRADE, E.T.: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – review & editing; CORREA, J.L.G.: Conceptualization, Data curation, Formal Analysis, Methodology, Software, Supervision, Validation, Visualization; MAGALHÃES, R.R.: Data curation, Formal Analysis, Software, Visualization.

DECLARATION OF INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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